ISSN 0001-4370, Oceanology, 2015, Vol. 55, No. 1, pp. 1–10. © Pleiades Publishing, Inc., 2015.
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**MARINE PHYSICS**

# Role of the Barotropic Water Exchange in the Formation of the Baltic Sea Level Spectrum

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Received November 8, 2013; in final form, February 7, 2014

**Abstract**—The spectrum of low-frequency sea level variations was analyzed on the basis of long-term (15–124 years) time series of sea-level data from three tide gauge stations in the Baltic Sea and two stations in the North Sea. The principal periodicities revealed in the spectrum are seasonal as well as the tidal oscillations including the pole tide with a period of about 14 months. The response function of the Baltic Sea level to variations in the North Sea level is calculated using a cross spectral analysis. It is shown that the barotropic water exchange through the Danish Straits is a basic factor in the formation of the low-frequency sea level spectrum in the Baltic Sea. The limited throughput of these straits plays the role of a natural low-pass filter for the sea level variations: high frequency sea level variations from the North Sea are effectively damped, while the low frequency signal can pass through into the Baltic Sea almost undisturbed. The simple model of the barotropic water exchange used in the study enables us to estimate the parameters of this filter. It is shown that the cut-off frequency is about 0.014 cpd (a period of 74 days): the energy of the sea level oscillations at this frequency is reduced by one half after their penetration into the Baltic Sea.

**DOI:** 10.1134/S0001437015010117

## 1. INTRODUCTION

The Baltic Sea is an inland sea. Its area is 393 000 km$^2$, the mean depth is 54 m, and the maximal depth is 459 m. It is connected with the North Sea through the narrow and shallow Danish Straits. The flow rate through these straits attains values higher than the river runoff by about 20 times [18]. This barotropic water exchange is actually a key factor that forms the low-frequency sea level variability. Rapid variations in the sea level in the Kattegat hardly penetrate into the Baltic Sea through the narrow and shallow straits—they are damped due to the wave energy dissipation in a turbulent flow. The system of straits is a low-pass filter [8], which successfully damps high-frequency sea-level oscillations, while slow (low-frequency) oscillations pass through into the Baltic Sea almost undisturbed.

The division of the sea-level variations in the Baltic Sea into two types has been suggested in [17]: (1) external oscillations induced by outside sea-level variations (in the Kattegat and the North Sea) and the influx of fresh water into the water area as a result of river run; and (2) internal oscillations caused by the effects of the variable air pressure, wind, and density on the Baltic Sea surface. The internal oscillations determine the sea level spectrum with periods shorter than 2 days, which are essentially connected with free natural oscillations (seiches) in the Baltic Basin. Variations in the sea level forced by external factors predominate in the spectrum of sea-level oscillations with periods longer than 1 month.

Results of the spectral analysis of sea level variations in the Baltic are presented in [3] on the basis of data on long-term hourly observations at 22 sites on the coast of Russia, Finland, Lithuania, and Estonia. According to the definition [17], the frequency range of the sea-level variations corresponds to the internal oscillations. It is shown that the sea-level spectrum is mainly formed under the action of the variable fields of air pressure and wind on the sea surface. Frequency selective properties of sea water areas, which determine the spatial structure of sea oscillations and resonance effects [4], play an important role.

In turn, low-frequency sea level variations in the Baltic Sea are caused more by the *large-scale* atmosphere–ocean interaction in the Northern Atlantic. The regularities of this interaction and the physical causes of the low-frequency variability are considered in [2].

This work continues studies of sea-level variations in the Baltic Sea and presents the results of the study of the spectrum of low-frequency sea-level oscillations with periods from 2 days to 20 years defined as external. The effect of the water exchange between the Baltic and North Seas on the formation of this spectrum is of special interest.

We do not analyze secular variations in the Baltic Sea level (trends) in detail in this work. This problem is quite well studied in several modern works, e.g., [13, 16].

Characteristics of the data used

| No. | Station name | Latitude, deg N | Longitude, deg E | Country | Time period |
|---|---|---|---|---|---|
| 1 | Cuxhaven | 53.87 | 8.72 | Germany | 1917–1987 |
| 2 | Göteborg | 57.68 | 11.79 | Sweden | 1967–2006 |
| 3 | Stockholm | 59.32 | 18.08 | Sweden | 1889–2012 |
| 4 | Gorniy Institute | 59.93 | 30.28 | Russia | 1977–2006 |
| 5 | Wladyslawowo | 54.80 | 18.42 | Poland | 1992–2006 |

## 2. DATA AND MAIN CHARACTERISTICS OF SEA-LEVEL VARIATIONS

The study of the low-frequency sea-level oscillations requires the use of long-term observation series. There are 30 stations at the Baltic coast where the period of the observations exceeds 100 years. The period of observations at the Stockholm station is long and equal to 240 years. In this work, the sea level variability in the Baltic is analyzed on the basis of hourly measurements at five tide gauges (table and Fig. 1): the Gorny Institute station (St. Petersburg, Russia), the Göteborg and Stockholm stations (Sweden), the Cuxhaven station (Germany), and the Wladyslawowo station (Poland). The continuous hourly measurements at Stockholm number 124 years. The measurement series at the Gorny Institute and Wladyslawowo stations are relatively short: 30 and 15 years, respectively. The sea level variations in the Kattegat were analyzed from measurement data at Göteborg; the length of the series is 40 years. The Cuxhaven station is external for the Baltic Basin; it is located at the German coast of the North Sea. The length of the data series of this station is 81 years. The data for the Cuxhaven, Göteborg, and Stockholm stations were taken from the University of Hawaii Sea Level Center (UHSLC) site (http://uhslc.soest.hawaii.edu). The sea level values measured were reduced to the Coordinated Universal Time, while the sea level, to zero of the Baltic System of Heights (0 BSH). The data were carefully verified for errors and faults; lags in records were filled with

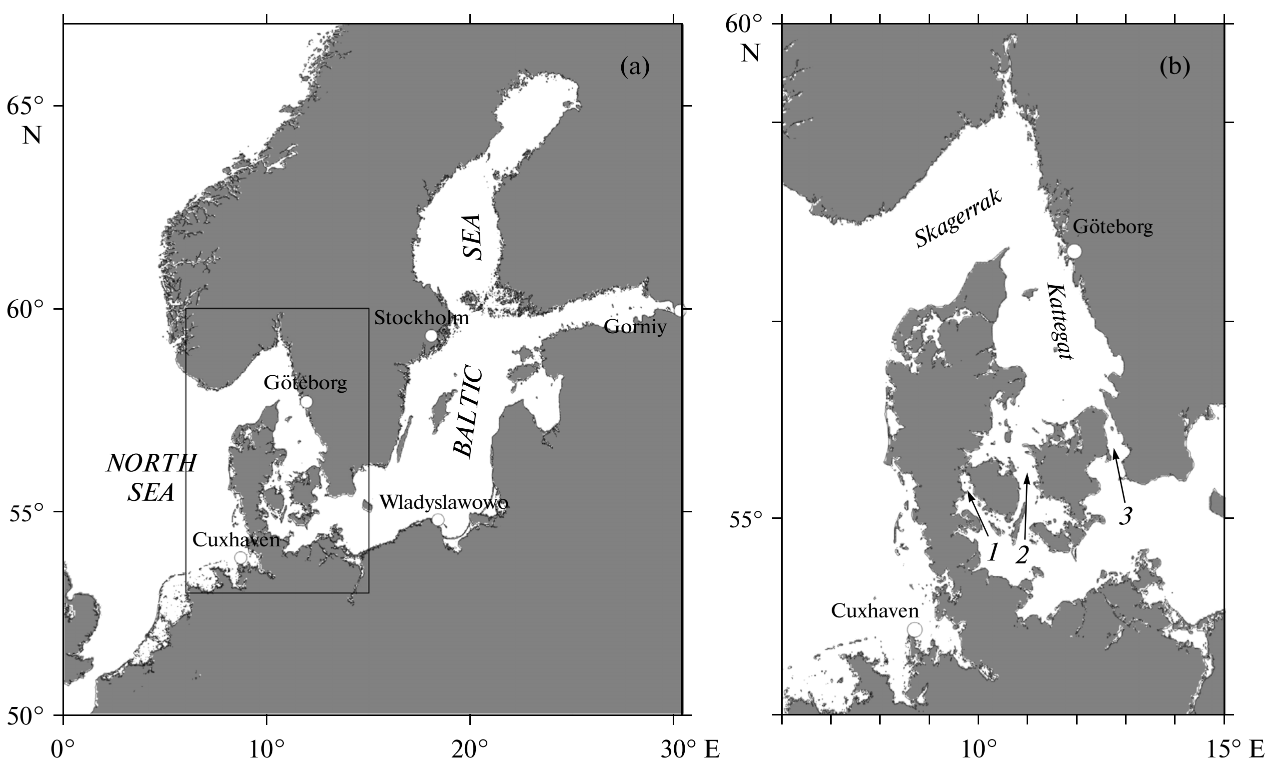


**Fig. 1.** (a) Map of the Baltic and North seas. Five stations with long-term sea level observations are shown, the data from which were used in the analysis. (b) Map of the Danish Straits (Small Belt (*1*), Great Belt (*2*), and Sound (*3*)).

interpolated values. The high quality of the data used in our analysis should be noted. Thus, the percentage of lags in the measurement data in the time periods under study did not exceed 1% at all the stations.

The Baltic Sea level permanently varies on different time scales. Figure 2 shows sea level variations plotted on the basis of data recorded in Stockholm with different sampling and averaging intervals.

Significant vertical displacements of the earth's crust are observed in the Baltic Sea; therefore, an unidirectional trend is characteristic of the interannual sea level variability in the Baltic Sea at most stations (Fig. 2a). The highest speeds of the crustal recoil are observed in the northern part of the Gulf of Bothnia (i.e., the sea level sharply drops). Crustal subsidence is observed in the southern part of the Baltic Sea, due to which the sea level variations have a positive trend. After subtraction of this trend from a record, the interannual sea level variability takes the form of a superposition of many different waves, where seasonal oscillations of the sea level prevail (Fig. 2b). They are expressed via the annual and semiannual components in the Baltic Sea. The seasonal components are of meteorological origin; hence, their amplitudes and phases noticeably vary from year to year. No clearly pronounced periodic components are observed in the sea level oscillations in the synoptic range of hydrometeorological processes (with periods of 2–30 days); the sea level oscillations are forced by meteorological factors (air pressure and wind stress) and are mainly random in character (Fig. 2c). A regular component is also seen, i.e., the seasonal variation in the sea level. Storm tides and seiches attain the highest amplitudes (up to 3–4 m) in the Baltic Sea, which occur against regular periodic oscillations—tides (Fig. 2d).

## 3. SPECTRAL ANALYSIS OF SEA LEVEL RECORDS

The spectral analysis of random processes is usually used for estimation of the frequency distribution of oscillation energy. Depending on the oscillation nature, the spectrum can have the character of a continuous distribution of energy over the frequency (continuum), which is typical for turbulent noise processes, or sharp deltalike peaks (discrete spectrum) corresponding to regular harmonic components with fixed frequencies. For example, tidal oscillations of the sea level are shown in the spectrum as deltalike peaks with frequencies of the fundamental tidal harmonics ($K_1$, $O_1$, $M_2$, $S_2$, et al.). In turn, the sea level variations caused by the effects of variable air pressure and wind on the sea surface are mainly random noises and have a spectrum in the form of a continuous function of the frequency.

The results of the spectral analysis of the sea level records from the data of the hourly measurements are shown in the frequency range from $10^{-4}$ to 0.5 cpd. For the more complete representation of the spectral dependence for such a wide frequency range, the spectrum was calculated with the use of the FFT algorithm of a whole record and with further averaging over the frequency of the periodogram calculated. The smoothing was carried out using a sliding Gaussian window, the width $\Delta f$ of which increases with the frequency $f$ so that the relative resolution $\Delta f/f = 0.05$ remains constant for all the frequencies. Figure 3 shows the spectra of the two longest sea level observation series from the Göteborg (1979–2006) and Stockholm (1889–2012) stations. The frequency dependence of the spectrum $S(f)$ is shown on a linear logarithmic scale, and the value of $fS(f)$ correctly shows the distribution of the sea level oscillation energy over the logarithmic frequency.

The energy (dispersion) of the low-frequency sea level oscillations is distributed mainly in the range of $10^{-3}$ to $10^{-1}$ cpd. When neglecting the seasonal peaks in the seal level spectrum in Göteborg, the maximal part of the energy falls in a synoptic range of periods of 5–50 days; most part of the sea level oscillation energy in Stockholm is accumulated in the annual range of periods from 30 to 300 days. The total oscillation energy is 505 cm$^2$ and 395 cm$^2$, respectively. This difference in the spectra inside and outside the Baltic Sea corresponds to the effect of the low-pass filtering mentioned above.

The vertical dashed lines in Fig. 3 show the frequencies that correspond to the main long-period tidal components: the 18.6-year component (Mn), the pole tide (14 months) (P14), the solar annual and semiannual components (Sa and Saa), the monthly moon component (Mm), the fortnightly moon component (Mf), and the moon-solar fortnightly tidal component (Msf).

The seasonal component plays a significant part in the total oscillation energy. In addition to the main Sa peak, the semiannual Ssa component is clearly pronounced in Figs. 3a and 3b. The long-term (>100 years) averaged amplitudes of the annual and semiannual components attain 14 and 5 cm, respectively, in the Baltic Sea. Maximal amplitudes of the annual component are observed in the Gulf of Bothnia and the Gulf of Finland, while those of the semiannual component, in the central part of the Baltic Sea (Stockholm) [10]. The annual sea level variations prevail in Göteborg, while the semiannual variations are much weaker.

The seasonal sea level variations are mainly connected with the annual cycle of variations in the air pressure gradients and wind fields, while with those of the temperature and water salinity to a lesser degree. The part of the gravitational tide in the seasonal sea level cycle is negligible; its amplitude is close to a static response and does not exceed several millimeters for the annual component.

It is well known that the tides in the Baltic Sea are formed due to the penetration of tidal waves from the North Sea and natural tidal waves induced immediately in the sea basin [14]. As shown in [6], the noticeable prevalence of the amplitudes of the daily harmon-

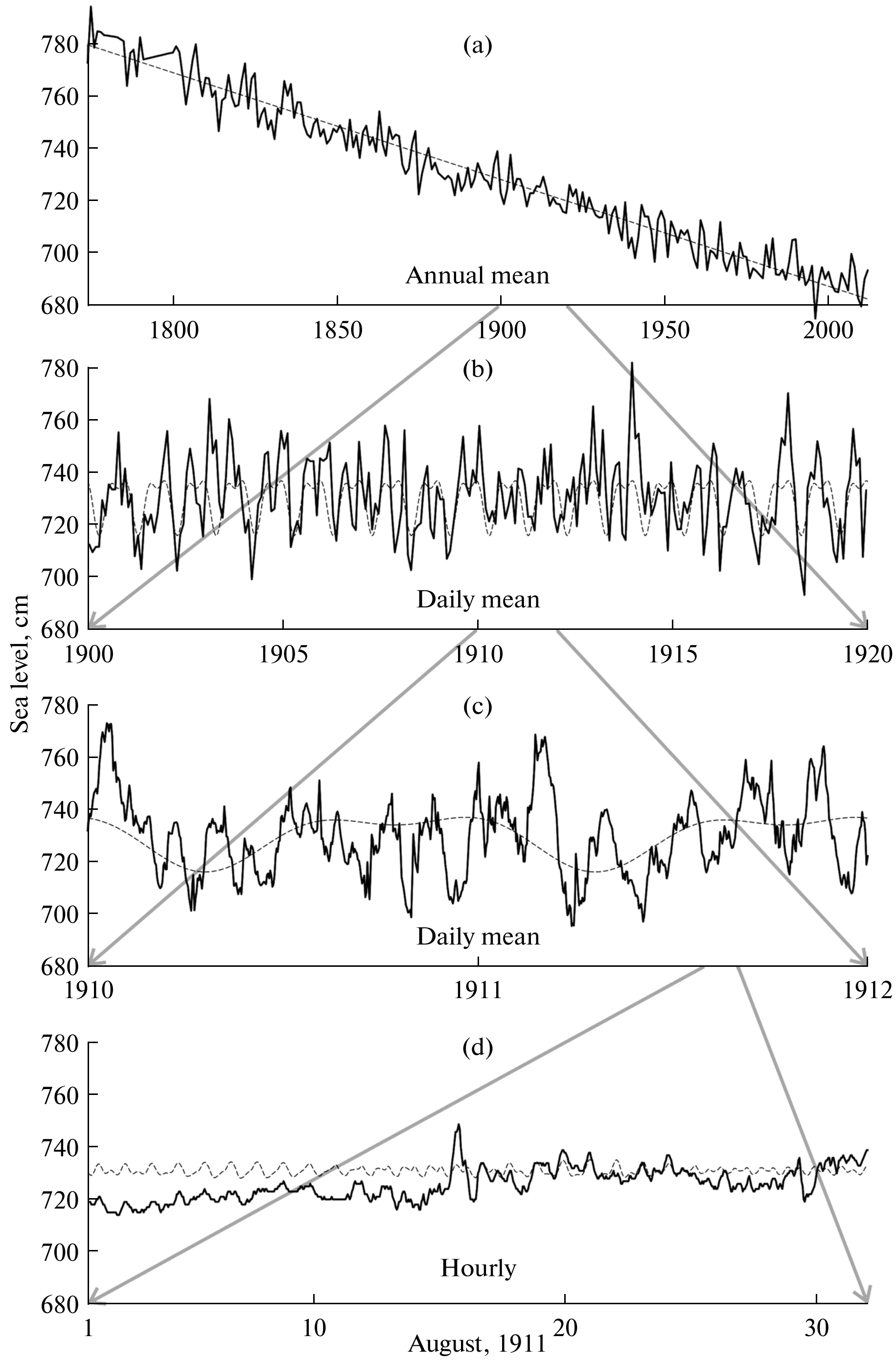


**Fig. 2.** Sea level records in Stockholm: (a) annual means, the dashed line shows the linear trend; (b) monthly mean, the dashed curve shows the long-term average seasonal variation; (c) daily mean, the dashed curve shows the long-term average seasonal variation; and (d) hourly, the dashed curve shows the tide calculated.

ics over the semidiurnal ones is typical for the tidal oscillations in the Baltic Sea. This feature seems surprising, since semidiurnal tides significantly prevail in the Atlantic Ocean and marginal seas. The tidal height is generally small (5–8 cm) and attains maxima (up to 20 cm) in the head of the Gulf of Finland [6]. The tidal sea level oscillations are much stronger in the North Sea; the spring tides attain a height of 3.2 m at the Cuxhaven station.

No significant peaks corresponding to long-period tidal components Mm, Msf, and Mf are distinguishable in the spectra of the sea level series under analysis.

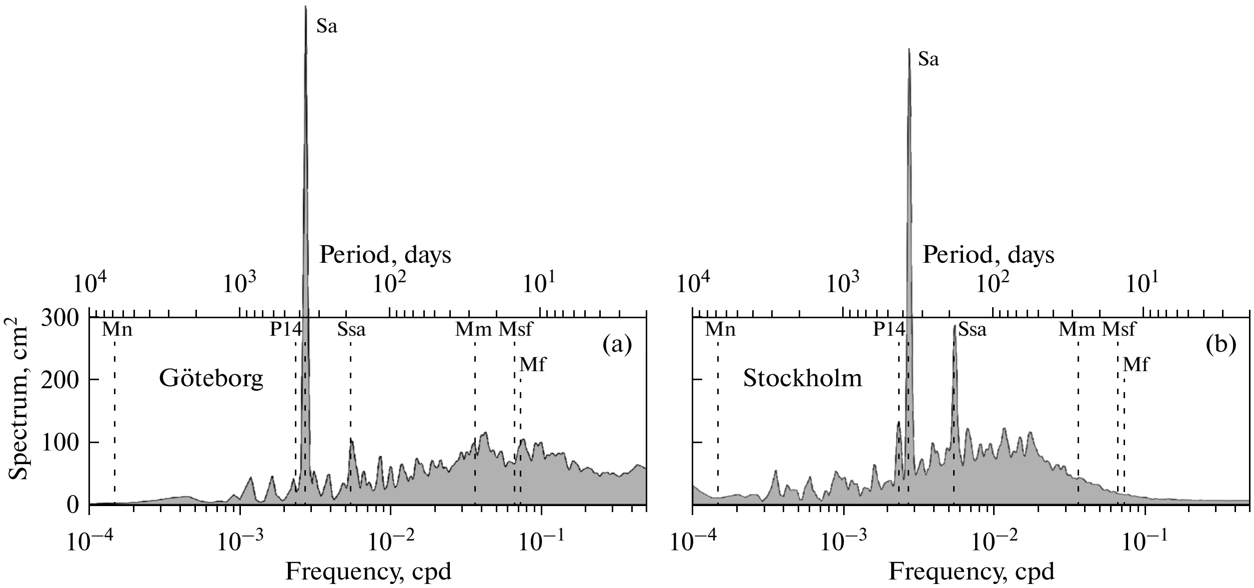


**Fig. 3.** Sea level oscillation spectra $fS(f)$ in (a) Göteborg (the North Sea, Sweden) and (b) Stockholm (the Baltic Sea, Sweden). The dashed lines show the frequencies of the tidal harmonics: the nodal 18.6-year (Mn), the monthly (Mm), the fortnightly (Msf and Mf), the seasonal (annual Sa and semiannual Ssa), and the 14-month pole tide (P14).

The amplitudes of these harmonics are quite small and hardly distinguishable against the noise in the Baltic Sea with spectral methods. It is also difficult to distinguish the long-period tidal harmonics Mn with a period of 18.6 years (nodal tide) in the spectra. Its amplitude has been assessed to be 0.6–0.9 cm in [19], which is close to the value of the equilibrium tide.

A peak with a frequency of about 0.86 cpy (period of 14 months) is seen in the sea level spectrum in Stockholm (Fig. 3b) calculated from a 124-year series. This harmonic corresponds to the Chandler wobble frequency, i.e., of natural nutation of the Earth's axis [5]. A corresponding wave in the ocean, similar to the long-period tidal oscillations in many aspects, was called the **pole tide** by G. Darwin [9]. The main features of the pole tide in the Baltic Sea were found in [7] on the basis of long-term sea level measurements. The spatial variability of the pole tide was described on the basis of the integral amplitudes calculated for 71 stations. The spectral analysis of long observation series revealed time variations in the oscillation amplitude and period. In general, the pole tide in the Baltic Sea is abnormally high. The pronounced increase in the tide amplitudes in the northeastward direction from 1.5 to 4.5 cm was noted in the Baltic Basin with maxima in the Gulf of Bothnia and the Gulf of Finland.

## 4. BAROTROPIC WATER EXCHANGE BETWEEN THE BALTIC AND NORTH SEAS AND ITS EFFECT ON THE FORMATION OF THE SEA LEVEL SPECTRUM

The water exchange between the Baltic and North Sea through the Danish straits is a key factor in the formation of the low-frequency sea level variability [18]. Long continuous flows that occur from time to time in narrow straits (Little Belt, Great Belt, and Sound) cause the significant migration of the water mass and can result in significant variations in the Baltic Sea volume. As has been stated in [8], the mean sea level in the Baltic can change by about a meter in less than a month due to intense water inflow–outflow through the Danish straits. The characteristic flow rate is about $10^5$ $m^3/s$ [11] in this case with peaks up to $10^6$ $m^3/s$.

The mismatch between the sea levels inside and outside the Baltic Sea occurs due to different causes: the tidal motion of the water mass, the wind or air pressure gradients, the river runoff, the precipitation, etc. The equalizing flow that originates in narrow and shallow straits is mainly controlled by two physical components: the friction force (hydraulic resistance) in the turbulent bottom boundary layer and the hydrostatic pressure gradient formed along the channel. The low throughput of the Danish Straits is a natural factor that isolates the Baltic Basin from the action of short-period sea level variations in the Kattegat. For example, tidal oscillations with amplitudes more than 1 m in the North Sea hardly penetrate in the Baltic Basin, where they are no more than several centimeters on the average. At the same time, the seasonal and interannual sea level variations in the North and Baltic Seas differ more weakly. Figure 4 shows synchronous records of the daily mean sea level values in Cuxhaven and Stockholm. It is seen that the sea level curve for Stockholm is similar to the smoothed curve for Cuxhaven. The low-pass filtering effect is seen in Fig. 4: sea level variations with periods shorter than 1 month, well seen in the North Sea (Cuxhaven), almost vanish

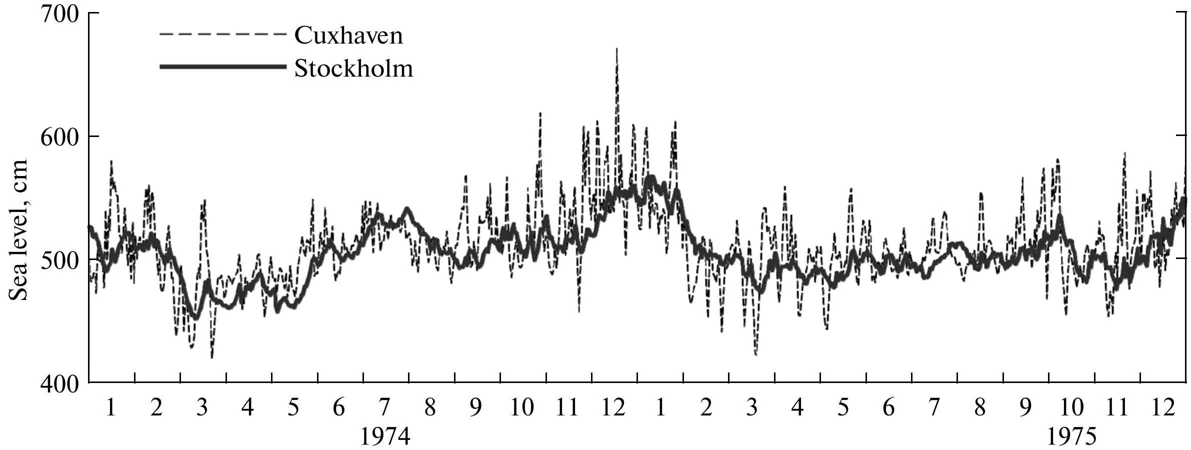


**Fig. 4.** Synchronous records of the daily mean sea level in Cuxhaven (the North Sea) and Stockholm (the Baltic Sea).

after passing the Danish straits and are invisible in the Stockholm record.

To calculate the parameters of this natural filter, let us estimate the spectral frequency characteristics that connect the sea level records in the North and Baltic Seas. We have selected the following pairs of synchronous sea level records: Cuxhaven and Stockholm (1967–1987),Göteborg and Stockholm (1983–2006), Cuxhaven and Gorniy (1977–1987), and Baltic stations Stockholm and Wladyslawowo (1992–2006) for comparison. In the process of calculations of the cross-spectral characteristics, the sea level observation series at the stations (Cuxhaven and Göteborg) located in the North Sea were set as **input** signals with respect to the inner (Baltic) stations Stockholm and Gorny, the records from which were considered as **output** signal of the linear system under study. The spectral analysis was carried out with the use of the FFT and a technique for smoothing the spectral density over segments with a Kaiser–Bessel window. The window length was set equal to 16384 hours, i.e., about 2 years. The number of degrees of freedom was varied from 18 to 48 for the pairs of records selected.

Figure 5 shows the cross-spectral characteristics calculated: the coherence, frequency response, and phase spectrum for the four pairs of sea level records (Figs. 5a–5d). The coherence function and the 95% confidence interval are shown in the upper row; the normalized frequency response and phase are shown below. The coherence is quite high at low frequencies, while the relatedness rating of the sea level oscillations remains poorly statistically assured for periods shorter than 5 days. That is, the so-called **external** (low-frequency) sea level oscillations in the North and Baltic seas correlate well; the low coherence for periods shorter than 5 days means that **internal** oscillations (e.g., seiches), which are not connected with sea level oscillations in the North Sea, play an important role in this frequency range in the spectrum. Let us note that the coherence remains quite high also at short periods for the pair Stockholm–Wladyslawowo, since both records relate to one basin.

The frequency response is an analog of the coefficient of regression of random parameters, and it specifies the measure of a coherent link between the input and output signals in a linear system within the theory of random processes: the ratio of the amplitude of an output signal harmonic (response) to the amplitude of an input signal of a specified frequency. One can see that the amplitude ratio varies with frequency similarly for three pairs of stations (Figs. 5a–5c): the (Baltic) sea level response to an external action (North Sea level variations) uniformly decreases as the frequency increases. The magnitude of the frequency response drops by an order at a frequency of 0.1 cpd. The phase functions calculated for these three pairs of stations (Figs. 5a–5c) show that the Baltic Sea level response noticeably lags behind the corresponding sea level variations in Göteborg and Cuxhaven. The phase shift is about –40° for periods of 100 days; i.e., variations in the Baltic Sea level lag behind North Sea level variations by more than 10 days. The phase is –90° (5 days) for periods of 20 days. Let us note that assessments of the frequency response and phase are less statistically significant (low coherence) at high frequencies (>0.1 cpd); therefore, their spread is much higher.

Low-frequency (<0.1 cpd) sea level oscillations are synchronous for the pair of Baltic stations Stockholm–Wladyslawowo. The Wladyslawowo/Stockholm response oscillates near 1. It means that slow sea level variations show an increase or decrease in the Baltic water volume.

It is evident that the frequency and phase characteristics calculated for the pairs of records in the North Sea–Baltic Sea system characterize the throughput of the Danish straits depending on the time scale of the sea level variability and the related water exchange.

Several simple models of the sea level response in partly isolated basins (bay, fiord, gulf, etc.) to the baro-

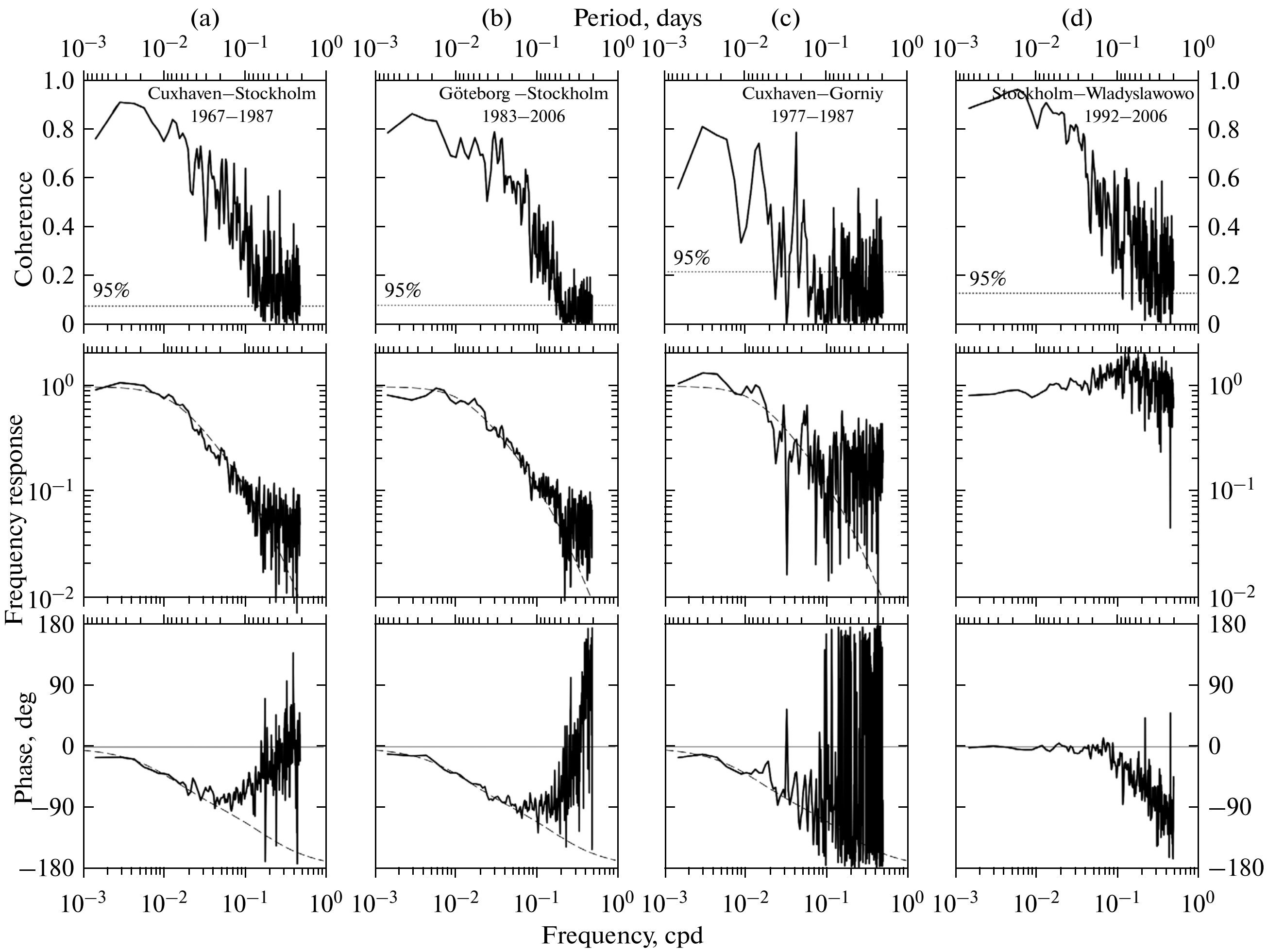


**Fig. 5.** Cross-spectral characteristics calculated for four pairs of synchronous records of the sea level: (a) Cuxhaven–Stockholm, (b) Göteborg –Stockholm, (c) Cuxhaven–Gorniy, and (d) Stockholm–Wladyslawowo. The upper row shows the coherence; the dashed line shows the 95% confidence interval. The middle row shows the module of the normalized frequency response, and the dashed curve shows the approximation of the variations in the response to analytical dependence (7). The bottom row shows the phase response, and the dashed curve shows the approximation of the variations in the phase to analytical dependence (8).

tropic effect from the open ocean are considered in [18]. To describe the correlation between the Baltic and North Sea levels, let us use the model that has been called the "Helmholtz resonator" by Stigebrandt. The Helmholtz seiche mode was first described by Miles in [15]. However, the Stigebrandt model additionally considers the energy dissipation due to turbulent friction in a channel.

Let us consider a basin (bay) of an arbitrary shape that is connected with an open sea by a narrow strait (see Fig. 6). Define the correlation between the water level in the bay and the variations in the level in the open sea within linear equations of motion of a homogeneous liquid in a shallow water approximation. Let us confine ourselves to considering "**slow**" motions of the liquid, during which the characteristic time scale of the sea level variations ($T$) significantly exceeds the travel time of a gravity wave through the basin area and the channel. It is reasonable to use the quasi-stationary approximation in this case, which is **hydraulic** in essence; i.e., the rate of change in the water level in the bay is connected with the tilt of the free surface along the channel, which, in turn, is determined by the flow rate and hydraulic resistance. The Coriolis force and the transverse tilt of the level can be ignored in a narrow strait. The geometry of the problem is shown in Fig. 6: the water basin area ($A$), the channel length ($L$), the depth ($D$), and the width ($W$). Therefore, the cross section area is $S = DW$.

Let consider the characteristic time of the water level adaptation in the bay to be much shorter than the typical time scale of variations in the flow rate in the channel. In this case, the level can be considered equal throughout the bay area. We should note that this assumption is debatable. When rotation is considered, the generation of gradient-vortex waves (e.g., shelf waves) is acceptable in the inner basin; they are much slower than gravity waves and can affect the rate of

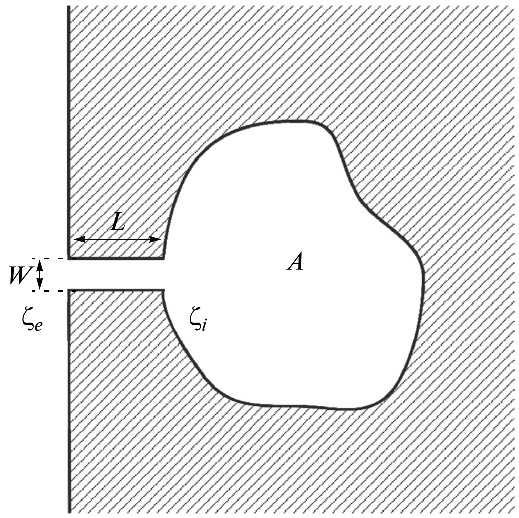


**Fig. 6.** Geometry of the problem (see the explanations in the text).

equilibration in the bay. The generation of gradient-vortex waves in the Baltic Sea was considered in [1] in detail. However, it is well known that these waves relate to the class of quasi-geostrophic motions, for which the level deviations are relatively small (solid cover approximation); therefore, this effect is neglected in the case under consideration.

We designate the level from the open sea side as $\zeta_e$ and the level in the bay as $\zeta_i$. The water balance equation is written as

$$A\frac{d\zeta_i}{dt} = uS, \tag{1}$$

where $u$ is the flow rate in the channel, and $t$ is the time. Let us note that the flow rate is considered constant throughout the whole channel. This condition is true if the channel length $L$ is much less than the length of a gravity wave with a period of about the time scale of the level variations ($L \ll T\sqrt{gD}$, where $g$ is the gravity acceleration).

The equation of liquid motion in the channel accounting for the linear hydraulic resistance can be written as

$$\frac{du}{dt} = -g\frac{d\zeta}{dx} - ru, \tag{2}$$

where $r$ is the resistance coefficient. Integrating Eq. (2) over the channel, we derive the hydraulic relationship that connects the tilt of the sea surface along the channel and the flow rate:

$$\frac{du}{dt} = g\frac{(\zeta_e - \zeta_i)}{L} - ru. \tag{3}$$

Let us note that Eq. (3) with neglect of the dissipation in the channel was used in the well known Limes model [15] for description of the Helmholtz seiche mode in a bay. To estimate the water exchange between the Baltic and North seas, Stigebrandt [18] suggested using an equation for calculation of the hydraulic resistance with a square dependence on the flow rate: the bottom friction force is defined as $\tau_b = C_b u^2$. Unfortunately, this nonlinear model is inconsistent with the concept of a linear system in the theory of random processes. It can be hardly used for the interpretation of results of the cross-spectral analysis of level observations (amplitude-phase frequency response). Therefore, we limit ourselves to the use of simple linear equation (3) for the calculation of the hydraulic resistance.

Equations (1) and (3) can be transformed into an ordinary differential equation that describes a damped oscillator:

$$\frac{d^2\zeta_i}{dt^2} + 2\delta\frac{d\zeta_i}{dt} + \Omega^2\zeta_i = \Omega^2\zeta_e, \tag{4}$$

where $\Omega^2 = \frac{gS}{AL}$ (here, $\Omega$ designates the natural frequency of the undamped oscillation), and $\delta = r/2$ is the damping coefficient.

We write Eq. (4) in a dimensionless form. For this, we substitute $\tau$ for $\Omega t$:

$$\frac{d^2\zeta_i}{d\tau^2} + 2\gamma\frac{d\zeta_i}{d\tau} + \zeta_i = \zeta_e, \tag{5}$$

where $\gamma = \frac{\delta}{\Omega}$ is the dimensionless damping coefficient.

Equation (5) defines a physical linear system that connects the input $\zeta_e$ and output $\zeta_i$ signals. The amplitude-frequency response of this system is well known; it is usually used for the description of the resonance effect.

Let us assume that a periodic change in the level $\zeta_e = a_e e^{i\omega t}$, $\omega = \frac{2\pi}{T}$, or $\zeta_e = a_e e^{i\sigma\tau}$ is observed from the outer side of the channel, where $\sigma = \omega/\Omega$ is the dimensionless frequency of the driving force. The forced solution has the form $\zeta_i = a_i e^{i\sigma\tau}$, where the correlation between the input and output level oscillation amplitudes is calculated from Eq. (5):

$$a_i = \frac{1}{(1-\sigma^2) + 2i\gamma\sigma}a_e = R(\sigma)e^{i\varphi}a_e, \tag{6}$$

where the frequency response moduluse can be written as

$$R(\sigma) = \frac{1}{\sqrt{\left(1-\sigma^2\right)^2 + 4\left(\gamma\sigma\right)^2}}, \tag{7}$$

and the phase is defined by the equation

$$\tan\varphi = \frac{2\gamma\sigma}{1-\sigma^2}. \tag{8}$$

The system behaves differently depending on the value of the damping coefficient: the resonance properties are shown only at $\gamma < 1$, the solution of homogeneous Eq. (6) has the character of damped oscillations, and the maximum of the frequency response corresponds to the resonance frequency $\sigma_0 = \sqrt{1-\gamma^2}$. Aperiodic damping is observed at $\gamma > 1$, and the so-called "critical damping," at $\gamma = 1$.

When this model is used for the Baltic Sea, the system of the Danish straits is assumed to be changed into one "equivalent" strait with the same flow rate. The form of the frequency responses and phase spectrum calculated (Fig. 5) correspond to aperiodic (supercritical) damping ($\gamma > 1$) of the linear system the North Sea—Baltic Sea. Approximating the amplitude (7) and phase (8) to the frequency response in the 0–0.1 cpd frequency range, the estimate of the damping coefficient $\gamma \approx 2.0$ can be found with the natural frequency of the undamped oscillations $\Omega \approx 0.31$ rad/day (period of about 20 days). Dependence (7) is actually a transfer function of the low-pass filter mentioned in [8]. The cut-off frequency of this filter is 0.014 cpd (period of 74 days): the energy of the sea level oscillations with this frequency is reduced by one half after their penetration into the Baltic Sea.

The most important result of the use of this model is that it allows calculation of the water discharge through the Danish straits through the difference in the mean levels of the Baltic and North Seas. A quasi-stationary approach was used in [18] under the assumption that the flow is controlled only by the difference between the water levels in the Kattegat and the southwestern part of the Baltic Sea. This actually means smallness of the acceleration $\frac{du}{dt}$ in Eq. (3). Neglecting the "dynamics," the flow rate in the channel can be written as $u \approx g\frac{(\zeta_e - \zeta_i)}{rL}$, and the equation for estimation of the flow rate $Q$ has the form

$$Q \approx \frac{(\zeta_e - \zeta_i)}{2\gamma}\Omega A. \tag{9}$$

According to the Göteborg and Stockholm data, the typical daily mean difference in the sea level $\sqrt{\langle(\zeta_e - \zeta_i)^2\rangle} \approx 0.3$ m, which corresponds to the mean flow rate through the straits calculated by Eq. (9), is $Q \approx 10^5$ m$^3$/s. This assessment completely coincides with the data of [12].

## 5. CONCLUSIONS

It is well-known that sea level variations mainly reflect barotropic liquid motions induced by tidal forces and actions of wind stress and air pressure gradients on the sea surface. River runoff, precipitation, and variations in the water density due to seasonal variations in the temperature and salinity can significantly affect the sea level in inner seas.

Division of the variations in the Baltic Sea level into two types was suggested in [17]: external and internal oscillations, meaning the sea level oscillations generated inside the Baltic Sea and the oscillations forced by outside variations in the sea level (in the North Sea). Just the external oscillations (low-frequency) became the subject of this study, which are mainly determined by the barotropic water exchange between the Baltic and North Seas.

The description of the low-frequency sea level spectrum in the Baltic assumes, first of all, the analysis of the energy distribution over the frequencies. The periodic components are pronounced in the spectrum in the frequency range from $10^{-1}$ to $10^{-4}$ cpd: the seasonal (annual Sa and semiannual Ssa) and the pole tide (P14) with a period of about 14 months.

The aperiodic low-frequency sea level oscillations are mainly caused by random effects of variable wind and air pressure on the sea surface. Intense water exchange through the Danish Straits can occur due to the different sea levels inside and outside the Baltic Sea. The limited throughput of these straits produces a natural low-pass filter. It was shown that oscillations of the North Sea level with a period of 10 days penetrate into the Baltic Sea 10-fold weakened. The cut-off frequency of the filter can be estimated as 0.014 cpd (period of 74 days); the energy damping coefficient for it is 1/2.

The simple theoretical model of the water exchange discussed in this work allows estimation of the water discharge through the Danish straits knowing only the difference in sea levels in the Kattegat and the Baltic. However, we should note the model limitation. This first concerns the assumption about the linear dependence of the hydraulic resistance in the strait on the flow rate. The possible difference in the air pressure at the observation stations was not taken into account as well when stating the model. It is evidently more reasonable to set a "reduced" level at different exits of the strait (i.e., the level minus the barometric response).

This simple approach to calculation of the water exchange can be successfully used for retrospective numerical simulation of variations in the Baltic Sea level.

## ACKNOWLEDGMENTS

This work was financially supported by the Russian Foundation for Basic Research (project nos. 12-05-00733, 13-05-41360, and 14-05-31461), Russian Science Foundation (project no. 14-37-00038) and the Ministry of Education and Science of the Russian Federation (project no. 11.G34.31.0007).

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*Translated by O. Ponomareva*