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A PROCEDURE FOR DETERMINING THE HEAT TRANSFER COEFFICIENTS OF SURFACES WITH REGULAR RELIEF

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A procedure for processing experimental data, which enables the fields of the distribution of the heat transfer coefficients on surfaces with regular relief to be determined for any temperature gradients and surface shapes is proposed. It is shown that, when estimating local and mean-integral characteristics of smooth surfaces a one-dimensional model of a semi-infinite body can be used, while in regions of considerable temperature gradients, particularly for curvilinear surfaces, the model gives reduced values of the heat transfer coefficient.
Keywords: *heat-exchange intensification, heat transfer coefficient, regular relief, holes.*

The problem of increasing the amount of heat that can be drawn off unit area of a surface for a specified temperature drop, i.e., the intensification of heat exchange, remains one of the most important at the present time. The increase in the number of publications, both Russian and abroad, and also the increasing complexity of the procedures for investigating these processes, indicates the importance of this subject. The action of the majority of known heat-exchange intensifiers is based on partial or complete rupture of the boundary layer of the gas (both thermal and dynamic) on the heat-exchange surface [1, 2]. As a rule, complex spatial vortex structures arise, and zones of flow recirculation are formed. Consequently, the distribution of the heat transfer and friction coefficients becomes exceptionally nonuniform [3]. Hence, for an experimental investigation of the thermal hydraulic characteristics of heat-exchange surfaces having a regular relief (i.e., distributed in accordance with a certain heat-exchange intensifier law), one must determine the local values of the parameters being investigated (the heat-exchange coefficient and the thermal resistance) with high spatial resolution and, if necessary, average these coefficients to obtain the mean-integral characteristics.

When determining the hydraulic characteristics, the use of direct methods of measurement gives the most reliable values of the overall resistance of the surface to the gas flow [4]. Obtaining the local heat transfer coefficients is considerably more difficult than the hydraulic characteristics, but the use of modern diagnostic and measuring instruments enables the characteristics of the heat exchange to be investigated with high spatial resolution. The most reliable methods of determining the local heat transfer coefficients and thermal characteristics of the surface are methods involving the use of liquid crystals and thermal imaging techniques, which enable one to investigate both steady and unsteady processes [5–7]. In this paper, the heat transfer coefficient was determined by an unsteady method using thermal imaging equipment.

The Experimental Bench and the Measuring Equipment. The experimental investigations were carried out in a small subsonic wind tunnel at the Institute of Mechanics of the Moscow State University [8]. The upper and lower walls of the slotted channel (height 30 mm, width 300 mm, and length 1080 mm) are sectioned. In one of the sections of the upper wall

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there is a Zn–Se window, transparent for infrared radiation and thus anabling the use of thermal-imaging equipment. On the lower wall of the channel, there is a working section, including two "floating" elements attached jointly with heaters on elastic elements and moving as a result of hydrodynamic interaction of the flow with the surface. Plates are placed on these elements, one of which is a sample plate with a smooth surface, while the other is the regular relief being investigated. Each of the sections of the lower wall can be heated, which enables investigations to be made for different lengths of both the thermal layer and the dynamic boundary layers. The temperature field nonuniformity within each section and between them does not exceed 2°C, while within the plate it is 1°C. The drop between the initial temperature of the wall surface and the temperature of the kernel of the flow is approximately 70°C, and the depth of cooling of the surfaces (the ratio of the temperature drops between the wall and the flow at the beginning and end of the experiment) reaches 0.3 [9]. This wall enables us to investigate the thermal and hydraulic characteristics of both surfaces simultaneously in the course of a single experiment.

We used the unsteady heat exchange method to determine the heat transfer coefficient: during the cooling of the surface of the wall being investigated in the course of approximately 40 sec we measured, at a fixed frequency (1 Hz) the temperature distribution on the surface of the plates using a thermal imaging camera and the temperature of the core of the flow using thermocouples. We then calculated the corresponding heat transfer coefficient.

Statement of the Problem. The most popular procedure for determining the heat transfer coefficient from the rate of cooling of the surface uses the solution of the one-dimensional heat conduction equation in a semi-infinite body in the form of the time dependence of the relative dimensionless surface temperature [9]:

$$\vartheta_n(\tau) = \frac{T_0 - T(0,\tau)}{T_0 - T_c} = 1 - e^{H^2 a\tau}\operatorname{erfc}\left(H\sqrt{a\tau}\right), \tag{1}$$

where T_0 is the surface temperature at the initial instant of time; $T(0, \tau)$ is the surface temperature at the instant of time τ; T_c is the temperature at the core of the flow; $H = \alpha/a$; α is the heat transfer coefficient; and a is the thermal diffusivity.

The assumptions made when solving (1) mean that this expression has limited applicability when processing experimental data. Consequently, it is necessary to set up a mathematical model which describes the processes occurring in the experiment more accurately. It is worth noting the following properties of one-dimensional model (1), which is difficult to use in experimental investigations.

1. The heating arrangement employed does not guarantee the uniformity of the temperature field at the initial instant of time. A heater placed between the elastic element and the model being investigated has a higher temperature than the surface considered.

2. The presence of local vortex structures and recirculation zones leads to nonuniformity of the heat-transfer processes in the boundary layer of the gas on the surface being investigated and, consequently, to a nonuniformity in the distribution of the coefficients α. This, in turn, gives rise to a nonuniformity of the temperature field on the surface when investigating a vortex-forming relief. The presence of such temperature gradients produces longitudinal and transverse thermal flows (with respect to the direction of motion of the gas).

3. Curvilinearity of the geometry of the heat-exchange surface.

4. The presence of a heat-exchange intensifier also leads to nonuniformity of the temperature fields and the thermal flows in the region of the greatest changes in the coefficient α.

In order to take into account the particular features of the surface geometry being investigated (in this investigation the holes) and the heat-exchange processes, which accompany the flow over the surface with such a relief (heat-exchange processes in separating zones and recirculation zones, and heat exchange when there are vortex structures) [1, 3], a method of determining the local coefficients α is necessary. For this purpose, in this research we used the solution of the differential equation of a three-dimensional unsteady process of the cooling of a plate of finite dimensions, obtained by the finite element method. We will demonstrate the main aspects of this procedure.

Consider the problem of the nonstationary propagation of heat in a solid with heat-exchange on the surface, described by the relation

$$q(x, y, \tau) = \alpha(x, y)\Delta T(x, y, \tau),$$

where $q(x, y, \tau)$ is the specific heat flux; x and y are the spatial coordinates (x coincides with the direction of the flow, and y is directed across the flow); $\Delta T(x, y, \tau) = T(x, y, \tau)|_S - T_c$ is the temperature head; and $S = S(x, y)$ is the limit of the heat exchange.

It is required to obtain the distribution of the coefficients α on the vortex-forming surface of the plate using the sets of temperatures on the surface of the plates and the temperatures in the kernel of the flow, obtained in the experiment, measured using two thermocouples, which are placed on the axis of the channel before and after the plate.

The Initial System of Equations, Boundary and Initial Conditions. Consider the equation of nonstationary heat conduction in an isotropic solid [9]:

$$\lambda(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2) = \rho c \partial T/\partial t, \tag{2}$$

where λ is the thermal conductivity, T is the temperature of the solid, ρ is the density, c is the specific heat capacity, and z is the spatial coordinate, directed along the normal to the OXY plane.

It is usually required to obtain the temperature distribution (field) $T = f(x, y, z, \tau)$ in the body, satisfying the initial conditions $T_0 = f_0(x, y, z, \tau)$ and the boundary conditions $T = T_b$ on the boundary S_1 and/or $\lambda(l_x \partial T/\partial x + l_y \partial T/\partial y + l_z \partial T/\partial z) + q + \alpha(T - T_c) = 0$ on S_2, where T_b is the known temperature of the boundary (the heat-exchange surface), and l_x, l_y, and l_z are the direction cosines of the vector normal to the boundary S.

To obtain the temperature field in the plate, the calculated region is divided into a certain number of finite elements, in which T is determined using the form functions of this element [10]. It was shown in [11] that when using linear elements (in this case the approximating function will be piecewise-continuous with a discontinuity of the first derivative on the boundaries of the elements), the problem of solving Eq. (2) is well-posed, and the solution can be obtained with a definite error.

In order to obtain the system of linear equations describing the temperature field in the region considered, we sum the following matrices for each element E:

the thermal conduction matrix

$$\mathbf{K}^{(E)} = \int_V \mathbf{B}^{\mathrm{T}} \mathbf{D} \mathbf{B} \, dV + \int_{s_2} \alpha \mathbf{N}^{\mathrm{T}} \, ds, \tag{3}$$

the heat-capacity matrix

$$\mathbf{c}^{(E)} = \int_V \rho c \mathbf{N}^{\mathrm{T}} \mathbf{N} \, dV, \tag{4}$$

the matrix of the column vector of the thermal load

$$\mathbf{f}^{(E)} = \int_V Q \mathbf{N}^{\mathrm{T}} \, dV + \int_{s_1} q \mathbf{N}^{\mathrm{T}} \, ds - \int_{s_2} \alpha T_c \mathbf{N}^{\mathrm{T}} \, ds, \tag{5}$$

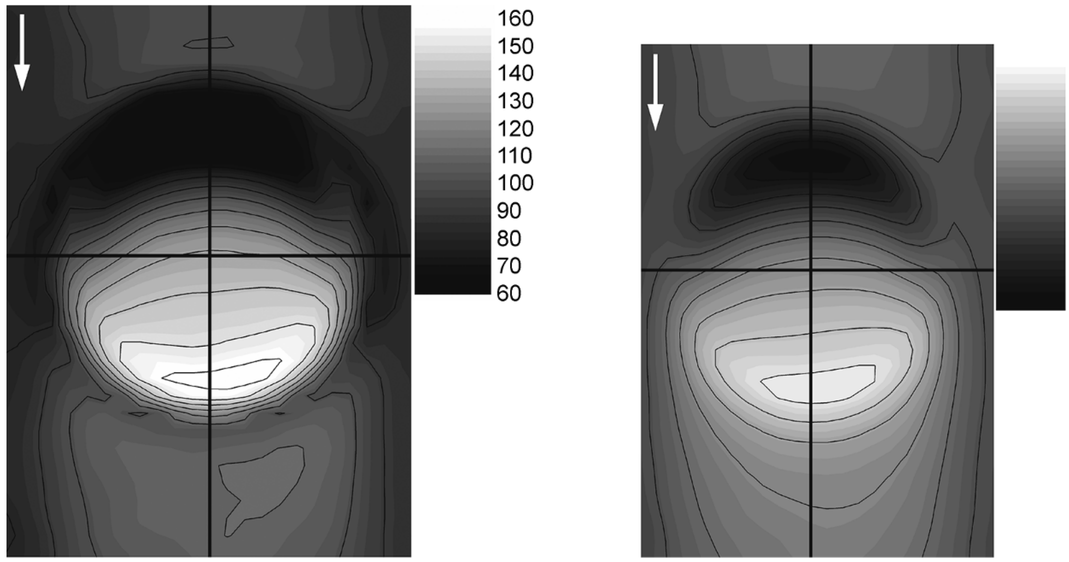
where $\mathbf{B}$ is the matrix of the derivatives of the shape functions of the element; $\mathbf{B}_{ij} = \partial \mathbf{N}_i / \partial x_j$, $i = 1, ..., n$, $j = 1, 2, 3$, n is the number of nodes of the finite elements; $\mathbf{D}$ is the diagonal matrix of the thermal conductivities; V is the volume of finite elements; $\mathbf{N}$ is a column vector of the shape function of the finite elements; s is the surface of the finite elements, and the surfaces s_1 and s_2 belong to the corresponding boundaries S_1 and S_2.

The integrals of Eqs. (3)–(5) are found for each finite element and collected in global matrices of the thermal conductivity, the thermal capacity and the thermal load, respectively. The result of the assembly is a system of first-order linear algebraic equations, which can be solved by any known method [11]:

$$\mathbf{C} \partial \mathbf{T}/\partial t + \mathbf{K}\mathbf{T} + \mathbf{F} = 0, \tag{6}$$

where $\mathbf{C}$ and $\mathbf{K}$ are the global heat capacity and thermal conductivity matrices, respectively, $\mathbf{F}$ is the column vector of the thermal load, and $\mathbf{T}$ is the column of values of the temperature and specified nodes.

The solution of (2) in the case considered is the temperature distribution at the nodes of the net, describing the plate in question. After setting up the global matrix, it is necessary to obtain the nodal values of the temperature in the plate and the



a *b*

Fig. 1. Distribution of the heat transfer coefficients α, W/(m^2·K), obtained using the three-dimensional model, ignoring (*a*) and taking into account (*b*) the nonlinearity of the surface for a plate with holes (the direction of the flow is indicated by the arrow).

boundary conditions on the plate surface – the distribution of the coefficients α. Terms defined by the coefficient α occur in the thermal conductivity matrix in the form of an additional term $\int_{s_2} \alpha \mathbf{N}^{\mathrm{T}} ds$ (here the values of the temperature at the nodes are known from experiment), and also in the vector of the boundary conditions $\int_{s_2} \alpha T_{\mathrm{c}} \mathbf{N}^{\mathrm{T}} ds$ (in our case, the temperature of the kernel of the flow T_{c} is known), where the temperature of the flow is constant for all points on the surface within the limits of a single time step. Hence, the number of unknowns in the system of equations obtained (the temperatures at the nodes, situated inside the plate, and the coefficients α occurring in the boundary conditions on the plate surface) is equal to the number of equations in system (6). The distribution of the coefficients α is determined together with the temperature field inside the plate by solving the system of equations obtained.

The Conditions for Carrying Out the Experiment, and the Time and Spatial Resolution of the Equipment. When the plate is heated using an electric heater (the backing) to the initial surface temperature T_0, the thermal field in the plate will not be uniform due to leakage of heat from its surface. Hence, the initial state of the plate for processing the experimental data is determined by solving Eq. (2) for the case when the plate is heated with the boundary condition of a constant heat flux from the backing [12].

The whole time interval of the cooling process is divided into equal steps $\Delta\tau = 1$ sec. However, when there are considerable temperature drops during a single time step, it is possible to divide the time interval between successive thermograms into several intermediate steps. Then, at the end of each step the temperature field on the surface should correspond to the temperature field on the thermogram.

The resolution of the thermal imaging system is 320 × 240 points, whereas the number of nodes along the *x* and *y* coordinates, and also in the depth of the plate (along the *z* coordinate), may be different, and, in general, it is not equal to the number of points where the temperature is recorded, which fall within the calculated region. In order to relate the values at the nodes of the net and the experimental data, a linear approximation of the temperature between the experimental points is used.

An Example of the Processing of the Experimental Data and a Comparison of the Results with the One-Dimensional Model. We investigated experimentally the heat exchange on a smooth surface and a surface with holes

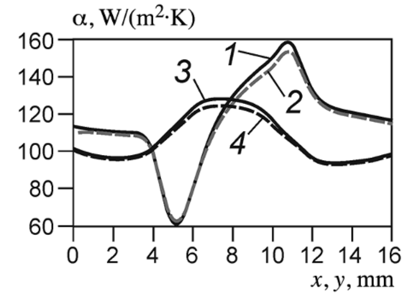


Fig. 2. Distribution of α, W/(m^2·K), along the longitudinal and transverse axes of the hole, obtained taking into account the nonlinearity of the surface for a plate: *1*, *2*) on the axis of the hole along the direction of the flow for the three-dimensional and one-dimensional [13] models, respectively; *3*, *4*) transverse to the direction of the flow.

(the longitudinal length of the plate was 125 mm, the transverse dimension was 100 mm, the thickness was 6 mm, and the material was plexiglass), the front edges of which were fixed in a slotted channel in parallel at a distance x = 350 mm from the input. The whole of the lower wall was heated, and consequently the length of the thermal boundary layer is 350 mm. The geometrical parameters of the array of holes were as follows: the hole depth was 1 mm and its diameter was 7.5 mm, the steps were 18 mm in the longitudinal direction and 12 mm in the transverse direction. The holes were arranged in a corridor. The parameters of the free stream remain constant during the experiment: its velocity was 59 m/sec, and the temperature of the core of the flow was 20.6°C.

We will consider the results of processing the experimental data using the procedure considered and the procedure [13] for determining the coefficients α for the region around a hole situated at the epicenter of the developed vortex flow on the surface being investigated in the middle (along the width of the plate) series of holes. The thermal imaging fields describing the region considered have a resolution of 43 × 43 points.

For a plate with holes, we show in Fig. 1*a* the distribution of the coefficients α obtained using the three-dimensional model, ignoring the nonlinearity of the surface. It follows from a comparison of the distributions of the coefficients α along the longitudinal and transverse axes of the hole, obtained using the one-dimensional and three-dimensional models, that the values obtained for a large part of the fragment of the plate considered agree with the exception of regions with considerable temperature gradients. Consequently, the choice of plexiglass with a low thermal diffusivity a as the material of the working plates enables one to use the one-dimensional model for processing the experimental data in the case of smooth surfaces without having to take into account the plate thickness (for the duration of the experiment considered [8]). The absence of relief enables thermal flows to be neglected in directions different from the normal to the surface. Then, the disagreement in the average coefficients α for the one-dimensional and three-dimensional plane models amounts to 1.5% and increases as these coefficients increase. At the same time, the instrumental errors when measuring the thermal characteristics of surfaces have an error when determining α of the order of 4% (for similar temperature drops and durations of the experiment) [13]. Hence, neglecting thermal fluxes in planes parallel to OXY, one can obtain errors not exceeding the errors in obtaining the experimental data.

The distribution of the coefficients α for a plate with holes, obtained taking the three-dimensional form of the heat fluxes and the curvilinearity of the geometry of the surface into account, is shown in Fig. 1*b*. In Fig. 2, we show the distributions of the coefficients α along the longitudinal and transverse axes of the hole, obtained using the one-dimensional and three-dimensional models. It follows from Fig. 2 that the curvilinearity of the surface makes a considerable contribution to the reproduction pattern of the heat fluxes. In regions corresponding to the bottom and the far edge of the hole (with respect to the flow), the values obtained using these procedures differ considerably. The coefficients α for the region where there is a break in the flow (along the far edge), calculated from (2), exceed the values obtained for the one-dimensional model by more than 13%, while the average values over the surface of the hole differ by 10%, which exceeds the errors in determining them. Over the plane part of the fragment of the plate, the coefficients α calculated using the one-dimensional and three-dimensional models agree with a sufficient degree of accuracy and, consequently, the coefficient α, averaged over the whole region, obtained taking the curvilinearity of the surface into account exceeds the corresponding value for the model of a semi-infinite body by 6%.

Conclusion. We have considered a procedure for determining experimentally the field of the heat transfer coefficients on surfaces with regular relief. The heat transfer coefficient was calculated using a three-dimensional equation of unsteady heat conduction, set up taking into account the experimentally obtained set of temperature distributions when the surface of the plate is cooled.

We have shown that the use of materials with low thermal conductivity enables one to neglect not only the nonuniformity of the initial temperature field in the plate, but also the overflow of heat in the longitudinal and transverse directions of the surfaces having a shape close to rectilinear. When solving such problems, it is best to use the one-dimensional model of a semi-infinite body (1). However, when considering the surface relief (in this case, holes) considerable differences were found between the model which takes into account the surface relief and the model of a semi-infinite body, particularly, in the region of considerable changes in the temperature and thickness of the plate. When the curvilinearity of the relief is taken into account, the value of the heat transfer coefficient averaged over the surface of a hole is 10% higher, which is particularly important when investigating the effect of the shape of the relief (the depression) on the thermal-hydraulic characteristics of the surfaces. Then, the heat transfer coefficient averaged over the whole of the region considered exceeds the similar value for the one-dimensional model by 6%. When using the three-dimensional model, ignoring the curvilinearity, the difference of the average heat transfer coefficient from the value obtained using the one-dimensional model does not exceed 1.5%.

Hence, in the experimental determination of both local and average values of the heat-exchange parameters, it is necessary to take the surface relief into account and, consequently, to use the corresponding processing procedure.

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