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Optimal Upsetting of Composite Cylinders

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Abstract—The upsetting of a composite cylinder in frictionless creep is analyzed in energy terms. Three loading programs that lead to upsetting of the cylinder by the same amount in the same time are compared. Variational analysis indicates that the kinematic loading program is best for industrial use.

Keywords: composite cylinder, upsetting, creep, energy-based analysis, variational calculus
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As a rule, normal temperatures are used for industrial processes such as upsetting, stamping, and pressing. Hot treatment is used to reduce the resistance of metals to irreversible deformation.

Numerous researchers have conducted tests of hollow and nonhollow cylinders with different geometry at high temperatures (up to 1200°C); see [1–9], for example. The equations of rigid–plastic deformation or creep theory are generally used in describing the test results. A cycle of problems regarding the upsetting of hollow and nonhollow cylinders in creep with different boundary conditions was solved in a classic monograph [10].

The basic results of research on the upsetting of hollow and nonhollow cylinders in creep at the Institute of Mechanics, Lomonosov Moscow State University, were presented in [11]. Topics considered include the upsetting of nonhollow cylinders, when barrel distortion is disregarded or taken into account; creep of hollow cylinders in free and constrained conditions; and the influence of friction on upsetting. Attention focuses on the energy consumed in the upsetting of cylinders with different loading programs. In addition to the theoretical research, the upsetting of nonhollow cylinders in high-temperature creep is considered experimentally. A special optical system reveals gradual barrel distortion of the cylinders in the course of upsetting. The theoretical and experimental results for the components of the creep-strain tensor are in good agreement.

Various upsetting programs for a uniform cylinder were investigated in energy terms in [12]. In the present work, we analyze the results obtained for a composite cylinder.

Consider the combination of a nonhollow inner cylinder A (radius R_{10}) and a hollow outer cylinder B (radii R_{10} and R_{20}). The cylinder height is $2H_0$ (Fig. 1). We assume the absence of friction between the bases of the cylinders and the plates of the press. Accordingly, no barrel distortion of the cylinders is observed in the course of upsetting in creep. For cylinders A and B, we assume steady creep with equal exponents n and different coefficients

$$\begin{cases} A\colon\ \dot{p}_u = \dfrac{1}{t_{01}}\left(\dfrac{\sigma_u}{\sigma_0}\right)^n; \\ B\colon\ \dot{p}_u = \dfrac{1}{t_{02}}\left(\dfrac{\sigma_u}{\sigma_0}\right)^n; \quad k = \dfrac{t_{02}}{t_{01}}, \end{cases} \tag{1}$$

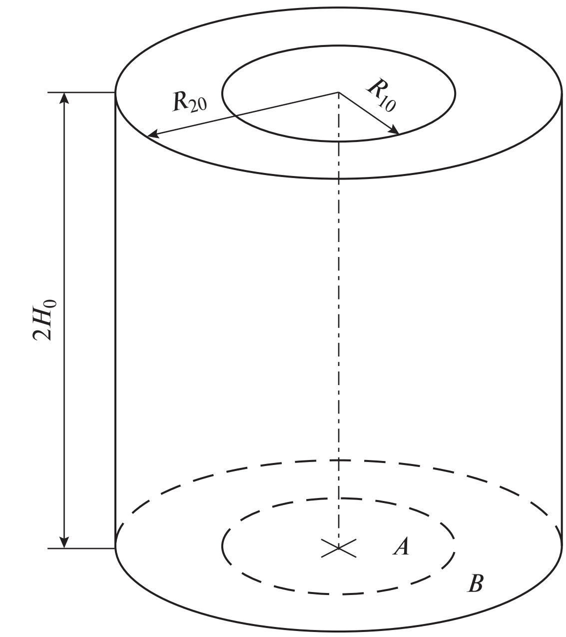

Fig. 1. Composite cylinder.

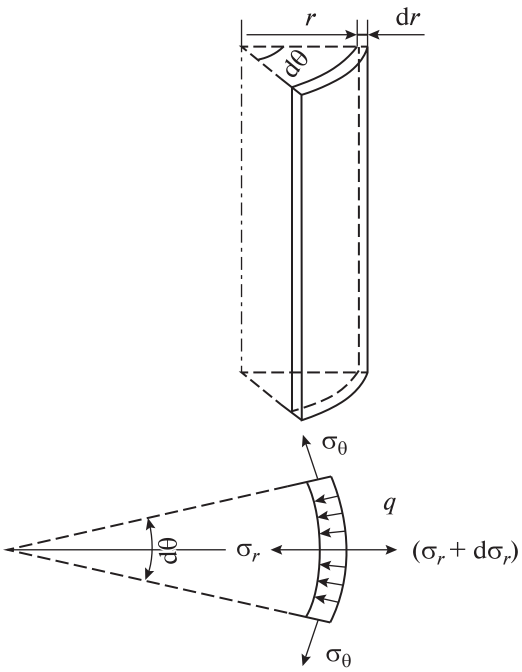

Fig. 2. Formulating the equilibrium equation of an element cut from a cylinder.

where σ_u and $\dot{p}_u$ are the effective stress and strain rate in creep; σ_0, t_{01}, t_{02} are constants.

The axial creep strain p_z is assumed to be the logarithm of the ratio of the height H to its initial value H_0

$$p_z = \ln\frac{H}{H_0}, \quad \text{that is} \quad \dot{p}_z = \frac{\dot{H}}{H} = -\frac{w}{H}. \tag{2}$$

Here $2w(t)$ is the rate at which the ends of the cylinder move closer together in upsetting. A point over a symbol denotes differentiation with respect to the time t.

The radial $\dot{p}_z$ and transverse $\dot{p}_\theta$ strain rates in creep are determined from the formulas

$$\dot{p}_r = \frac{\mathrm{d}u}{\mathrm{d}r}; \quad \dot{p}_\theta = \frac{u}{r},$$

where r is the radius; and $u(r,t)$ is the radial velocity of the points of the cylinder.

The incompressibility condition

$$\dot{p}_r + \dot{p}_\theta + \dot{p}_z = \frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r} - \frac{w}{H} = 0$$

implies that

$$u = \frac{wr}{2H} + \frac{C(t)}{r}.$$

From the boundary condition $u(r = 0, t) = 0$, we find that $C(t) \equiv 0$. Then we may write

$$u = \frac{wr}{2H}; \quad \dot{p}_r = \dot{p}_\theta = \frac{w}{2H}. \tag{3}$$

Taking account of Eqs. (2) and (3), we determine the strain rate in creep

$$\dot{p}_u = \frac{\sqrt{2}}{3}\left[(\dot{p}_r - \dot{p}_\theta)^2 + (\dot{p}_\theta - \dot{p}_z)^2 + (\dot{p}_z - \dot{p}_r)^2\right]^{0.5} = \frac{w}{H}.$$

Using Eq. (1), we may determine the effective stress in the inner cylinder A and outer cylinder B

$$\begin{cases} A\colon\ \sigma_u = \sigma_0 t_0^{1/n}\left(\dfrac{w}{H}\right)^{1/n}; \\ B\colon\ \sigma_u = \sigma_0 t_0^{1/n}\left(\dfrac{w}{H}\right)^{1/n} k^{1/n}. \end{cases}$$

The relation between the components of the stress tensor σ_{ij} and strain-rate tensor $\dot{p}_{ij}$ takes the form

$$\dot{p}_{ij} = \frac{3}{2}\frac{\dot{p}_u}{\sigma_u}\left[\sigma_{ij} - \frac{(\sigma_r + \sigma_\theta + \sigma_z)}{3}\delta_{ij}\right]; \quad i,\ j = r,\ \theta,\ z, \tag{4}$$

where δ_{ij} is the Kronecker delta.

From Eq. (4)

$$(\sigma_z - \sigma_r) = \frac{2}{3}\frac{\sigma_u}{\dot{p}_u}(\dot{p}_z - \dot{p}_r) = -\frac{2}{3}\sigma_u\left(\frac{w}{H}\right)^{-1}\frac{3}{2}\frac{w}{H} = -\sigma_u.$$

According to Eq. (3), the strain rates $\dot{p}_r$ and $\dot{p}_\theta$ in creep are equal. Consequently, it follows from Eq. (4) that

$$\sigma_r = \sigma_\theta. \tag{5}$$

From the incompressibility condition, we determine the relation between the radial coordinates of the cylinder in the initial state (r_0) and after upsetting (r)

$$\begin{cases} \dfrac{r}{r_0} = \left(\dfrac{H_0}{H}\right)^{0.5}; \\ R_1 = R_{10}\left(\dfrac{H_0}{H}\right)^{0.5}; \\ R_2 = \left(\dfrac{H_0}{H}\right)^{0.5}. \end{cases} \tag{6}$$

From the deformed cylinder, we cut two radial cross sections (corresponding to azimuthal angle 2φ) and two azimuthal cross sections with radii r and $(r + \mathrm{d}r)$, as illustrated in Fig. 2. The equilibrium equation of this section of the cylindrical shell takes the form

$$-\sigma_r r\mathrm{d}\varphi \times 2H + (\sigma_r + \mathrm{d}\sigma_r)(r + \mathrm{d}r)\mathrm{d}\varphi 2H - \sigma_\theta \mathrm{d}r \times 2H\mathrm{d}\varphi = 0,$$

Hence, taking account of Eq. (5), we obtain

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = 0.$$

It follows from the condition $\sigma_r(r = R_2, t) = 0$ that, with ideal slip of the cylinder's end surfaces relative to the press plates, the radial and transverse stresses are identically zero

$$\sigma_r(r) \equiv \sigma_\theta(r) \equiv 0.$$

Only the longitudinal stress σ_z is nonzero

$$\begin{cases} (\sigma_z)_A = (\sigma_r)_A - (\sigma_u)_A = -\left(\dfrac{w}{H}\right)^{1/n}; \\ (\sigma_z)_B = (\sigma_r)_B - (\sigma_u)_B = -k^{1/n}\left(\dfrac{w}{H}\right)^{1/n}. \end{cases}$$

This solution is valid if the effective frictional force $q = -\mu\sigma_z$ where μ is the frictional coefficient, is no greater than the maximum tangential stress $\tau_{\max}$, which may be expressed approximately in terms of the effective stress [13]

$$\tau_{\max} = \left(2/\left(2+\sqrt{3}\right)\right)\sigma_u = 0.535\sigma_u.$$

If $q > \tau_{\max}$, then the effective frictional force at the contact surface of the cylinder and the press plates must be replaced by the maximum tangential stress. Given that $q = 0$ in the absence of friction, while $\tau_{\max} > 0$, we conclude that $q < \tau_{\max}$.

We now introduce the dimensionless variables

$$\bar{t} = \frac{t}{t_0};\ \bar{H} = \frac{H}{H_0};\ a = \frac{R_{10}}{R_{20}};\ \bar{r} = \frac{r}{R_{20}};$$
$$\bar{R}_1 = \frac{R_1}{R_{20}};\ \bar{R}_2 = \frac{R_2}{R_{20}};\ \bar{u} = \frac{t_{01}}{H_0}u;\ \bar{w} = \frac{t_{01}}{H_0}w;$$
$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\sigma_0};\ \bar{P} = \frac{P}{\pi R_{20}^2\sigma_0};\ \bar{V} = \frac{V}{\pi R_{20}^2 H_0\sigma_0},$$

where P is the external compressive force; and V is the work of force P in axial motion of the ends of the cylinder. (In what follows, the bar over the symbols will be omitted.)

We now calculate compressive force P as a function of the height H. Taking account of Eq. (6), we find that

$$\begin{aligned} P &= -\int_0^{R_1}(\sigma_z)_A r\mathrm{d}r - \int_{R_1}^{R_2}(\sigma_z)_B r\mathrm{d}r \\ &= \left(\frac{w}{H}\right)^{1/n}\frac{R_1^2}{2} + k^{1/n}\left(\frac{w}{H}\right)^{1/n}\frac{\left(R_2^2 - R_1^2\right)}{2} \\ &= K(w)^{1/n}(H)^{-\frac{n+1}{n}}; \\ K &= \frac{1}{2}\left[a^2 + k^{1/n}\left(1-a^2\right)\right]. \end{aligned} \tag{7}$$

We consider the upsetting of a cylinder with height $H(t=0) = 1$ to $H_1(t=t_1) = H_1$ at time $t = t_1$ with three different loading programs. So as to be specific, we assume the following values: $H_1 = 0.9$ when $t_1 = 0.5$ and $H_1 = 0.8$ when $t_1 = 1.0$. We will compare the work V of the external compressive force P in moving the ends of the cylinder

$$V = \int_{H_1}^{1} P\mathrm{d}H$$

and select the program corresponding to minimum V.

Note that an energy-based approach has been adopted in the analysis of other mechanical processes for a deformable solid. For example, the irreversible deformation of metals in which the specified strain is attained after a specific time with minimum energy consumption was considered in [14]. The corresponding experimental deformation conditions were determined.

According to the kinematic loading program, constant rate $w(t) = w_0$ is substituted into Eq. (7). In that case

$$\begin{aligned} P(H) &= K(w_0)^{1/n} H^{-\frac{n+1}{n}}; \\ V_1 &= \int_{H_1}^{1} P(H)\mathrm{d}H = Kn(w_0)^{1/n}\left[H_1^{-\frac{1}{n}} - 1\right]. \end{aligned} \tag{8}$$

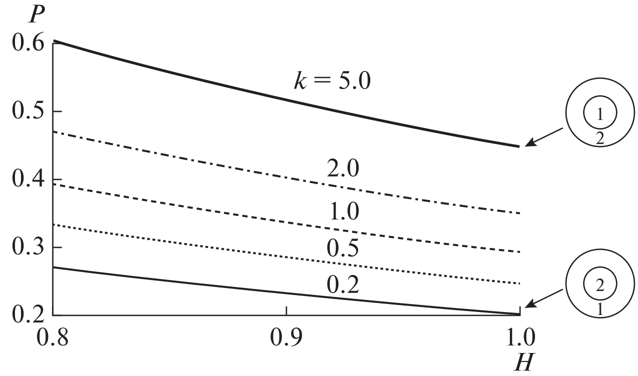

Fig. 3. Dependence of the external force P on the cylinder height H in the kinematic loading program when $w_0 = 0.2$, $n = 3$, and $a = 0.5$: cylinder *1* is soft, and cylinder *2* is hard.

In Fig. 3, we plot $P(H)$ when $w_0 = (1 - H_1)/t_1 = 0.2$, $n = 3$, and $a = 0.5$ for different values of k. We see that, if the outer hollow cylinder is more rigid than the inner nonhollow cylinder according to Eq. (1) — that is, if $k > 1$ — the necessary upsetting force P is greater than when $0 < k < 1$.

In the upsetting of a cylinder with a mechanical loading program, we need to determine the constant

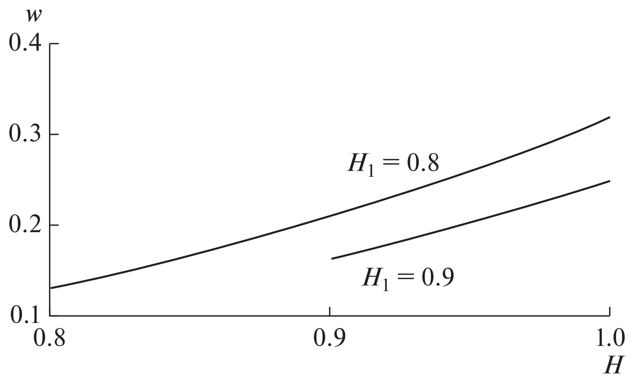

Fig. 4. Dependence of the speed w on the cylinder height H in the mechanical loading program when $n = 3$ and $a = 0.5$.

upsetting force P_0 such that the cylinder height reaches H_1 at time $t = t_1$. From Eq. (7)

$$\begin{cases} P_0 = K\left(-\dot{H}\right)^{1/n} H^{-\frac{n+1}{n}}; \\ -\dot{H} = \dfrac{\mathrm{d}H}{\mathrm{d}t} = \left[\dfrac{P_0}{K}\right]^n H^{n+1}; \\ t = \left(\dfrac{K}{P_0}\right)^n \displaystyle\int_H^1 H^{-(n+1)}\mathrm{d}H. \end{cases} \tag{9}$$

From Eq. (9), we may determine the constant upsetting force P_0 such that the cylinder height $H(t = t_1) = H_1$, the velocity w, and the work V_2 of force P_0 as follows

$$\begin{cases} P_0 = K\left[\dfrac{H_1^{-n} - 1}{t_1 n}\right]^{1/n}; \\ w(H) = \dfrac{\left[H_1^n - 1\right]}{t_1 n} H^{(n+1)}; \\ V_2 = K(1 - H_1)\left[\dfrac{H_1^{-n} - 1}{t_1 n}\right]^{1/n}. \end{cases} \tag{10}$$

When $n = 3$, the constant upsetting force P_0 is $0.628K$ when $H_1 = 0.9$ and $t_1 = 0.5$; and $0.682K$ when $H_1 = 0.8$ and $t_1 = 1$. In Fig. 4, we plot $w(H)$ when $H_1 = 0.9$ and 0.8, $n = 3$, and $a = 0.5$. Note that w does not depend on K.

We now consider the optimal hybrid kinematic–mechanical loading program, corresponding to the minimum possible work V_3 of the external force P. The basic relations take the form

$$\begin{cases} H(t) = 1 - \displaystyle\int_0^t w\mathrm{d}t; \\ w = -\dot{H}; \\ P = K(w)^{1/n} H^{-\frac{n+1}{n}}; \\ V_3 = \displaystyle\int_{H_1}^1 P\mathrm{d}H = K\int_{H_1}^1 \left(-\dot{H}\right)^{1/n} H^{-\frac{n+1}{n}}\mathrm{d}H. \end{cases}$$

If the exponent n is expressed as the ratio of two odd numbers, the work $V_3(t)$ may be determined from the formula

$$V_3 = K\int_0^{t_1}\left(\frac{\dot{H}}{H}\right)^{\frac{n+1}{n}}\mathrm{d}t. \tag{11}$$

The necessary condition for an extremum of the functional $J = \int_0^{t_1} F\left(t, H(t), \dot{H}(t)\right)\mathrm{d}t$ is determined from the Euler equation [15]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial F}{\partial \dot{H}}\right) - \frac{\partial F}{\partial H} = 0,\quad H(0) = 1,\quad H(t_1) = H_1. \tag{12}$$

It follows from Eq. (12) that

$$F = K\left(\frac{\dot{H}}{H}\right)^{\frac{n+1}{n}};\quad \frac{\partial F}{\partial H} = -K\frac{(n+1)}{n} H^{-\frac{2n+1}{n}}\dot{H}^{\frac{n+1}{n}};$$

$$\frac{\partial F}{\partial \dot{H}} = K\frac{(n+1)}{n} H^{-\frac{n+1}{n}}\dot{H}^{\frac{1}{n}};$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial F}{\partial \dot{H}}\right) = K\frac{(n+1)}{n}$$

$$\times\left[\frac{1}{n} H^{-\frac{n+1}{n}}\dot{H}^{-\frac{n+1}{n}}\ddot{H} - \frac{(n+1)}{n} H^{-\frac{2n+1}{n}}\dot{H}^{\frac{n+1}{n}}\right].$$

Substituting our results into Eq. (12), we may write a second-order differential equation

$$\ddot{H} = (H)^{-1}\left(\dot{H}\right)^2,\quad H(t = 0) = 1,\quad H(t = t_1) = H_1. \tag{13}$$

To solve Eq. (13), we introduce the transformations

$$\dot{H} = \frac{\mathrm{d}H}{\mathrm{d}t} = s;\quad \ddot{H} = s\frac{\mathrm{d}s}{\mathrm{d}H} = \frac{s^2}{H};$$

$$\frac{\mathrm{d}s}{s} = \frac{\mathrm{d}H}{H};\quad s = \frac{\mathrm{d}H}{\mathrm{d}t} = C_1 H.$$

As a result, we obtain the dependence $H(t)$ with two constants of integration

$$t = \frac{1}{C_1}\ln H + \ln C_2.$$

Table 1. Work V_1, V_2, and V_3 of the external force P in the kinematic, mechanical, and optimal loading programs ($n = 3$, $a = 0.5$)

k	V_1 ($H_1 = 0.9$)	V_1 ($H_1 = 0.8$)	V_2 ($H_1 = 0.9$)	V_2 ($H_1 = 0.8$)	V_3 ($H_1 = 0.9$)	V_3 ($H_1 = 0.8$)
0.2	0.02159112	0.04664282	0.02162663	0.04698706	0.0215867	0.0465999
0.5	0.02650359	0.05725512	0.02654718	0.05767767	0.0264981	0.0572024
1.0	0.03135497	0.06773546	0.03140655	0.06823536	0.0313485	0.0676731
2.0	0.03746733	0.08093987	0.03752896	0.08153722	0.0374596	0.0808653
5.0	0.04805093	0.10380338	0.04812996	0.10456946	0.0480411	0.1037077

By the substitutions $H(t = 0) = 1$ and $H_1(t = t_1) = H_1$, we may write the solution of Eq. (13) in the form

$$H(t) = H_1^{t/t_1}. \tag{14}$$

Substituting Eq. (14) into Eq. (11), we find that

$$\begin{gathered} V_3 = K\int_0^{t_1}\left(\frac{1}{t_1}\ln H_1\right)^{\frac{n+1}{n}} \mathrm{d}t = K(\ln H_1)^{\frac{n+1}{n}} t_1^{-\frac{1}{n}}, \\ K = \frac{1}{2}\left[a^2 + k^{1/n}\left(1 - a^2\right)\right]. \end{gathered} \tag{15}$$

Table 1 presents the work V_1, V_2, and V_3 of the external compressive force corresponding to Eqs. (8), (10), and (15), respectively, when $H_1 = 0.9$ and 0.8, $n = 3$, and $a = 0.5$, with different values of k.

Table 2 presents values of four comparative characteristics of V_1, V_2, and V_3 for the same values of the height H_1 (at the corresponding t_1) and the exponent n

$$\begin{cases} \Delta_1 = \dfrac{(V_1 - V_3)}{V_3}; \\ \Delta_2 = \dfrac{(V_2 - V_3)}{V_3}; \\ \Delta_3 = \dfrac{(V_2 - V_3)}{(V_1 - V_3)} = \dfrac{\Delta_2}{\Delta_1}; \\ \Delta_4 = \dfrac{(V_2 - V_1)}{V_1} = \dfrac{\Delta_2 - \Delta_1}{1 + \Delta_1}. \end{cases}$$

Thus, Δ_1 characterizes the extent to which V_1 exceeds V_3; Δ_2 characterizes the extent to which V_2 exceeds V_3; Δ_3 compares the extent to which V_2 exceeds V_3 with the extent to which V_1 exceeds V_3; and Δ_4 characterizes the extent to which V_2 exceeds V_1.

For all the parameter values, V_2 slightly exceeds V_1, as is evident from Δ_4. In other words, the kinematic loading program is more effective than the mechanical program. This discrepancy increases with increase in n.

If we compare Δ_2 and Δ_1 in Table 2, we conclude that the amount by which V_2 exceeds V_3 is considerably greater (by a factor of 10–100, depending on n) than the amount by which V_1 exceeds V_3. When $3 \le n \le 11$, Δ_1 is only hundredths or thousandths of a percent. Since the values of V_1 and V_3 are very close, Δ_3 is relatively large: 9–120. Hence, in terms of energy consumption, the kinematic loading program is expedient for the industrial upsetting of composite cylinders.

Table 2. Results for the three loading programs

H_1	t_1	Δ_i	$n = 3$	$n = 5$	$n = 7$	$n = 9$	$n = 11$
0.9	0.5	Δ_1, %	0.0206	0.0111	0.0076	0.0057	0.0046
		Δ_2, %	0.19	0.28	0.37	0.46	0.55
		Δ_3	9.0	25.0	48.9	80.7	120.1
		Δ_4, %	0.16	0.27	0.36	0.45	0.55
0.8	1.0	Δ_1, %	0.092	0.050	0.034	0.026	0.021
		Δ_2, %	0.83	1.24	1.64	2.04	2.41
		Δ_3	9.0	24.9	48.5	79.5	117.3
		Δ_4, %	0.74	1.19	1.61	2.01	2.39

It follows from Eqs. (8), (10), and (15) that, with specified w_0, these factors depend only on H_1 (that is, on t_1) and n. They do not depend on a and k—that is, on K. Note also that our findings do not depend on the ratio of the cylinder's height and its transverse dimensions.

CONCLUSIONS

We have investigated the upsetting of a composite cylinder in creep, with different loading programs. The steady creep of the inner and outer cylinders is described by power laws with the same exponent and different values of the coefficients. We consider frictionless upsetting with no barrel distortion of the cylinders.

In energy terms, upsetting with constant speed of the compressive plates is found to be more effective than upsetting with constant loading force.

Variational analysis is used to determine the optimal loading program, with the minimum possible work of the external compressive force. The work in the optimal loading program differs by hundredths and thousandths of a percent from the work in the kinematic loading program. Therefore, the kinematic loading program is best for industrial use.

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REFERENCES

1. Tarnovskii, I.Ya., Levanov, A.N., and Poksevatkin, M.I., *Kontaktnye napryazheniya pri plasticheskoi deformatsii* (Contact Stresses in Plastic Deformation), Moscow: Metallurgiya, 1966.
2. Sivak, I.O., Ogorodnikov, V.A., Pekhov, G.F., and Syrnev, B.V., Calculation of the limiting shaping of blanks from a hardly-deforming alloy with axisymmetric upsetting, *Kuznechno-Shtampovochnoe Proizvod.*, 1980, no. 2, pp. 2–5.
3. Burov, Yu.G., Calculation of metal temperature of a billet during hot upsetting, *Kuznechno-Shtampovochnoe Proizvod.*, 1984, no. 11, p. 14.
4. Archagov, A.T. and Nekrasov, V.A., Analysis of torsional upsetting, *Kuznechno-Shtampovochnoe Proizvod.—Obrab. Mater. Davleniem*, 2003, no. 3, pp. 21–26.
5. Antoshchenkov, Yu.M. and Taupek, I.M., Specific modeling of axisymmetric upsetting, *Kuznechno-Shtampovochnoe Proizvod.—Obrab. Mater. Davleniem*, 2014, no. 10, pp. 42–49.
6. Lin, Z.C. and Lin, Z.C., An investigation of a coupled analysis of a thermo-elastic-plastic model during warm upsetting, *Int. J. Mach. Tools Manuf.*, 1990, vol. 30, no. 4, pp. 599–612.
7. Lin, S.Y., Investigation of the effect of dissimilar interface frictional properties on the process of hollow cylinder upsetting, *J. Mater. Process. Technol.*, 1997, vol. 66, nos. 1–3, pp. 204–215.
8. Thiebaut, C., Bonnet, C., and Morey, J.M., Evolution of the friction factor of a molybdenum work piece during upsetting tests at different temperatures, *J. Mater. Process. Technol.*, 1998, vol. 77, nos. 1–3, pp. 240–245.
9. Lin, S.Y. and Lin, F.C., Influences of the geometrical conditions of die and workpiece on the barreling formation during forging–extrusion process, *J. Mater. Process. Technol.*, 2003, vol. 140, pp. 54–58.
10. Malinin, N.N., *Polzuchest' v obrabotke metallov* (Creep of Metals during Processing), Moscow: Mashinostroenie, 1986.
11. Lokoshchenko, A.M., *Polzuchest' i dlitel'naya prochnost' metallov* (Creep and Prolonged Strength of Metals), Moscow: Fizmatlit, 2016.
12. Lokoshchenko, A.M., Optimal mode for upsetting of cylinders with friction, *Vestn. Mashinostr.*, 2016, no. 9, pp. 44–48.
13. Malinin, N.N., *Prikladnaya teoriya plastichnosti i polzuchesti* (Applied Theory of Plasticity and Creep), Moscow: Mashinostroenie, 1975.
14. Krotov, V.F. and Brovman, M.Ya., Extreme processes of plastic deformation of metals, *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. Mashinostr.*, 1962, no. 3, pp. 148–153.
15. Krasnov, M.L., Makarenko, G.I., and Kiselev, A.I., *Variatsionnoe ischislenie, zadachi i uprazhneniya* (Calculus of Variation, Tasks, and Exercises), Moscow: Nauka, 1973.

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