ISSN 0027-1349, Moscow University Physics Bulletin, 2013, Vol. 68, No. 2, pp. 114–117. © Allerton Press, Inc., 2013.
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The Electromagnetic Mechanism of Plasmon Decay to a Neutrino Pair in a Strongly Magnetized Electron Gas

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Received December 10, 2012

Abstract—An expression for the neutrino luminosity of a degenerate electron gas in a strong magnetic field via plasmon decay to a neutrino pair due to electromagnetic neutrino moments is derived. The neutrino luminosity of the medium in an electromagnetic reaction channel is shown to be comparable with the luminosity in a weak channel. The relative upper bounds for the effective magnetic neutrino moment are obtained.

Keywords: neutrino, neutrino magnetic moment, plasmon, neutron star, strong magnetic field, degenerate electron gas

DOI: 10.3103/S0027134913020021

INTRODUCTION

Neutrino emission is the key mechanism of the energy losses of stars in the late stage of their evolution [1]. We will consider cooling of the outer regions of neutrino stars that are rarefied enough to assume that they are transparent to originating neutrinos. Strong magnetic fields ($H \gtrsim 10^{12}$ G) can exist in these regions; moreover, the fields for the class of the neutron stars that are called magnetars can reach 10^{14}–10^{16} G [2] (see also [3]).

The key processes that lead to neutrino production in the outer regions of neutron stars are annihilation of $e^-e^+ \longrightarrow \nu\bar{\nu}$, photoproduction of a neutrino pair on the electron $\gamma e^{\pm} \longrightarrow e^{\pm}\nu\bar{\nu}$, photon decay $\gamma \longrightarrow \nu\bar{\nu}$, and two-photon annihilation $\gamma\gamma \longrightarrow \nu\bar{\nu}$. The key results of the study of these processes without a magnetic field were given in the review [4]. The luminosity of a degenerate nonrelativistic gas due to photoproduction of neutrino pairs for the case of a superstrong field was determined in [5]. The authors of [6] estimated the luminosity of the degenerate electron gas that was induced by these processes in a superstrong field (except electron–positron annihilation, whose contribution is negligible, owing to the smallness of the positron fraction). The results of photoproduction of neutrino pairs were refined in [7].

In a minimal standard model of the electroweak interaction, neutrinos are massless and do not possess electromagnetic dipole moments. Simple extension of the model yields the formation of a magnetic dipole moment (MDM) of a massive Dirac neutrino, which is determined by one-loop radiative corrections $\mu_\nu \simeq 3.2 \geq 10^{-19}\,(m_\nu/1\text{ eV})\mu_{\rm B}$ [8] (where m_ν is the neutrino mass and $\mu_{\rm B}$ is the Bohr magneton), which is several orders of magnitude lower than the existing laboratory, astrophysical, and cosmological bounds for μ_ν. However, significantly larger neutrino dipole moments are theoretically possible; they can lead to the effects that are observable in the laboratory and affect reactions with the emission of neutrino pairs in astrophysics, which are analogous to the reactions due to weal interactions.

The electromagnetic mechanism of the above processes (except two-photon annihilation) and neutrino bremsstrahlung on the nucleus $e^-(Ze) \longrightarrow e^-(Ze)\nu\bar{\nu}$ was studied in [9]. It is shown that at the known laboratory and cosmological bounds for the electromagnetic dipole moments of the neutrino the electromagnetic mechanism of neutrino pair emission can compete with a weak one.

Electromagnetic neutrino properties were discussed in the review [10] (see also [1]). The upper bounds for the electric d_ν and magnetic μ_ν dipole neutrino moments, which are obtained from astrophysical and cosmological considerations, are of the order of $(10^{-12}$–$10^{-10})\mu_{\rm B}$ and substantially depend on the models that are used (see [1], p. 627, and the references therein). In particular, the authors of [11] gave a conservative bound that was obtained from the analysis of solar neutrinos [12]

$$\mu_\nu < 0.54 \times 10^{-10}\mu_{\rm B}. \quad (1)$$

The bound that has been recently obtained in the GEMMA laboratory experiment on antineutrino scattering off electrons is as follows [13]:

$$\mu_\nu < 3.2 \times 10^{-11}\mu_{\rm B}. \quad (2)$$

The vertex operator of the photon–neutrino coupling (for the Dirac neutrino) is as follows in [14–16] (see also [1, 10])[1]

$$V^{\alpha}(k) = \mu_B \sigma^{\alpha\beta} k_{\beta}[f_{2\nu}(k^2) + i\gamma^5 g_{2\nu}(k^2)], \qquad (3)$$

where k is the 4-momentum of the photon and $\sigma^{\alpha\beta} = (\gamma^{\alpha}\gamma^{\beta} - \gamma^{\beta}\gamma^{\alpha})/2$. Hereinafter, taking the relative smallness of k^2 in the analyzed processes into account, we will use the static values of the electromagnetic neutrino form factors $f_{2\nu} = f_{2\nu}(0) = \mu_\nu/\mu_B$, $g_{2\nu} = g_{2\nu}(0) = d_\nu/\mu_B$.

1. PLASMON DECAY TO A NEUTRINO PAIR (ELECTROMAGNETIC MECHANISM)

We will consider a degenerate electron gas in a strong magnetic field H:

$$T \ll \mu - m, \quad H > (\mu^2 - m^2)/(2e), \qquad (4)$$

where T is the temperature, $\mu \simeq \mu(T = 0) \equiv \varepsilon_F = \sqrt{m^2 + p_F^2}$ is the chemical potential of the electron gas, m is the electron mass, and ε_F and p_F are the Fermi energy and momentum. These conditions being met, the electrons of the medium occupy only the ground Landau level (the principle quantum number is $n = 0$), and

$$p_F = 2\pi^2 n_e/(eH), \qquad (5)$$

where n_e is the concentration of electrons and the electron charge is $-e < 0$. In this case, determination of the dispersive properties of the photon in the medium (which will be needed below) is substantially simplified.

The photon dispersion in a strongly magnetized plasma was considered in detail in [17]. Under these conditions, the photons of two different polarizations are propagating (namely, modes *2* and *3* using the terminology of [17]). Their polarization vectors are as follows

$$\epsilon_{\alpha}^{(2)} = \frac{\tilde{F}_{\alpha\beta}k^{\beta}}{H\sqrt{k_{\parallel}^2}}, \quad \epsilon_{\alpha}^{(3)} = \frac{F_{\alpha\beta}k^{\beta}}{H\sqrt{k_{\perp}^2}}, \qquad (6)$$

where $F_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$ is the tensor of the external electromagnetic field and its dual tensor, $k^{\alpha} = (k_0, \mathbf{k})$ is the 4-momentum of the photon, and $k_{\perp}^2 = k_x^2 + k_y^2$, $k_{\parallel}^2 = k_0^2 - k_z^2$, $k^2 = k_{\parallel}^2 - k_{\perp}^2$. It is well known [5, 17] that in the case of a superstrong magnetic field it is enough to take the interaction of electrons only with the photons of mode *2* into account. The dispersion law for mode *2* at $k_0 \ll 2m$ is as follows

$$k^2 = k_0^2 - k_z^2 - k_{\perp}^2 = \omega_p^2, \qquad (7)$$

where ω_p is the plasma frequency [17]. It can be said that the photon acquires a nonzero mass. For conditions (4), it can be approximately estimated with the expression

$$\omega_p = k_0(\mathbf{k} = 0) = \left(\frac{2\alpha}{\pi}\frac{H}{H_0}\frac{p_F}{\varepsilon_F}\right)^{1/2} m, \qquad (8)$$

which can be derived from the general formula (3.2) on p. 96 in [17]. Here, $H_0 = m^2/e = 4.41 \times 10^{13}$ G.

The analyzed conditions also allow one to neglect the renormalization of the photon wave function ($\epsilon_{\alpha}^{(2)} \longrightarrow \sqrt{Z_2}\epsilon_{\alpha}^{(2)}$) in the magnetized medium ($\sqrt{Z_2} \simeq 1$) [6].

The general expression for the luminosity (the rate of energy losses by a unit volume of a medium) due to the process of the plasmon decay to a neutrino pair $\gamma \longrightarrow \nu\bar{\nu}$ through the electromagnetic channel reads as follows

$$Q_{em} = \int \frac{d^3k d^3q d^3q'}{2^3(2\pi)^9 k_0 q_0 q_0'} \times (2\pi)^4 \delta^{(4)}(q + q' - k)|M|^2 k_0 n_B(k_0), \qquad (9)$$

where $n_B(k_0) = (e^{k_0/T} - 1)^{-1}$ is the Bose distribution function for photons.

The matrix element of the process is written as

$$M = \epsilon_{\alpha}^{(2)}(\bar{u}_{\nu}(q')V^{\alpha}u_{\nu}(-q)), \qquad (10)$$

where $\bar{u}_{\nu}(q')$ and $u_{\nu}(-q)$ are the bispinors of the neutrino and antineutrino with 4-momenta q' and q, respectively, and its square is

$$|M|^2 = \epsilon_{\alpha}^{(2)}\epsilon_{\beta}^{(2)}J^{\alpha\beta}. \qquad (11)$$

Here, the neutrino trace is

$$J^{\alpha\beta} = \mathrm{tr}[\hat{q}'V^{\alpha}(q, q')\hat{q}\bar{V}^{\beta}(q, q')]. \qquad (12)$$

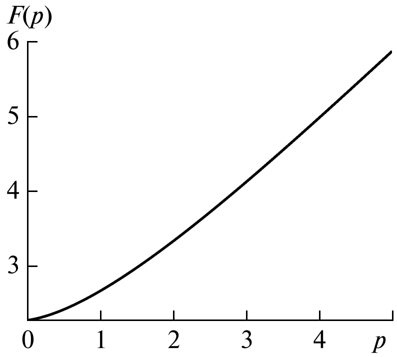
Here, as in [7], the approximation of massless neutrinos is used ($q'^2 = q^2 = 0$) where the cosmological bound for the sum of active (light) neutrino masses is taken into account [11]: $\sum m_{\nu} \lesssim 1$ eV.

Standard integration of Eq. (12) over the neutrino pair momenta gives

$$\int \frac{d^3q}{q_0}\frac{d^3q'}{q_0'}\delta^{(4)}(q + q' - k)J^{\alpha\beta} = \frac{4\pi k^2}{3}(k^{\alpha}k^{\beta} - g^{\alpha\beta}k^2)\bar{\mu}_{\nu}^2, \qquad (13)$$

where the effective magnetic moment $\bar{\mu}_{\nu}$ of the neutrino is introduced to characterize the neutrino-pho-

[1] The system of units is used where $\hbar = c = k_B = 1$, $\alpha = e^2/4\pi \simeq 1/137$, and the pseudo-Euclidean metric with the signature $(+ - - -)$; $\hat{a} = \gamma^{\beta}a_{\beta}$ is the convolution of the Dirac matrices γ^{β} with the 4-vector $a^{\beta} = (a^0, \mathbf{a})$; $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$.



ton interaction in accordance with the following expression

$$\bar{\mu}_\nu^2 = \mu_B^2(f_{2\nu}^2 + g_{2\nu}^2) = \mu_\nu^2 + d_\nu^2. \quad (14)$$

For neutrino luminosity (9), with Eqs. (11), (13), and (7) taken into account, we get

$$Q_{em} = \frac{\bar{\mu}_\nu^2 \omega_p^4}{48\pi^3} \int_0^\infty \frac{k^2 dk}{e^{\frac{1}{T}\sqrt{\omega_p^2 + k^2}} - 1}, \quad (15)$$

where $k = |\mathbf{k}|$.

The asymptotic behavior of luminosity (15) in two characteristic limiting cases is as follows. At $\omega_p \ll T$

$$Q_{em} = \frac{\zeta(3)}{24\pi^2} \alpha \hat{\mu}_\nu^2 \frac{\omega_p^4 T^3}{m^2}, \quad (16)$$

and at $\omega_p \gg T$

$$Q_{em} = \frac{\alpha \hat{\mu}_\nu^2}{3 \cdot 2^{9/2} \pi^{3/2}} \frac{\omega_p^{11/2} T^{3/2}}{m^2} e^{-\frac{\omega_p}{T}}, \quad (17)$$

where the reduced (in μ_B units) moment is (see Eq. (14))

$$\hat{\mu}_\nu = \bar{\mu}_\nu / \mu_B. \quad (18)$$

DISCUSSION OF THE RESULTS

The upper (relative) bound on $\hat{\mu}_\nu$ (18) will be found from the following requirement: the neutrino luminosity in the electromagnetic channel should be lower than that in a weak channel, namely $Q_{em} < Q_w$. Comparing Eq. (15) with the corresponding result from [6], we obtain

$$\hat{\mu}_\nu < \frac{G_F m}{\sqrt{2}\pi\alpha} T F(p). \quad (19)$$

Here, the function

$$F(p) = p\left[\bar{g}_V^2 + \frac{2}{3}\bar{g}_A^2 \frac{B_4(p)}{B_2(p)}\right]^{1/2} \quad (20)$$

of the argument $p = \omega_p/T$ is introduced; it is expressed through the integrals

$$B_n(p) = \int_0^\infty \frac{x^n dx}{\exp(p\sqrt{1+x^2}) - 1}$$

and the effective coupling constants

$$\bar{g}_V^2 = \sum_{l = e, \mu, \tau} g_V^2(l) \simeq 0.929,$$

$$\bar{g}_A^2 = \sum_{l = e, \mu, \tau} g_A^2(l) = 3/4.$$

Function (20) is a monotonically increasing one; it asymptotically approaches a linear function (see figure):

$$F(p) \geq F(0) = 2[2\zeta(5)/\zeta(3)]^{1/2} \bar{g}_A \simeq 2.275, \quad (21)$$

$$F(p) \simeq \bar{g}_V p \simeq 0.964p \quad \text{for} \quad p \gg 1. \quad (22)$$

Let us write Eq. (19), with Eq. (21) taken into account, in the form which is convenient for astrophysical applications

$$\hat{\mu}_\nu < 1.58 \times 10^{-12} T_8 F(p) \geq 3.60 \times 10^{-12} T_8. \quad (23)$$

Here,

$$p = \frac{\omega_p}{T} = \left(\frac{2\alpha}{\pi}\frac{H}{H_0}\right)\left[1 + \left(\frac{m}{p_F}\right)^2\right]^{-1/4} \frac{m}{T} = 1.92(1 + 0.44 H_{13}^2 \rho_6^{-2})^{-1/4} H_{13}^{1/2} T_8^{-1}, \quad (24)$$

where relations (8) and (5) are used, as well as the electron density under the conditions of the neutron star crust (see [18]) expressed through the density of the matter ρ and the proton mass m_p: $n_e \simeq 0.5\rho/m_p$. In addition, the following notations for the dimensionless quantities are introduced: $H_{13} = H/(10^{13}\text{ G})$, $T_8 = T/(10^8\text{ K})$, $\rho_6 = \rho/(10^6\text{ g/cm}^3)$.

For the case of $\omega_p \ll T$ ($p \ll 1$), we obtain from Eq. (23) (with Eq. (21) taken into account) as follows:

$$\hat{\mu}_\nu < 3.6 \times 10^{-12} T_8. \quad (25)$$

The condition $\omega_p \ll T$ for a degenerate electron gas holds true at relatively high temperatures. In particular, at $T_8 = 1.8$ we get the bound $\hat{\mu}_\nu < 6.5 \times 10^{-12}$, which is slightly weaker than that found in [7] ($\hat{\mu}_\nu < 1.1 \times 10^{-12}$) from the comparison of the electromagnetic and weak mechanisms of the photoproduction $\gamma e \longrightarrow e\nu\bar{\nu}$ [7], which turns out to be more effective under the same conditions than the process of the plasmon decay [6].

In the case of $\omega_p \gg T$, from Eq. (23) (with Eq. (21) taken into account) we obtain

$$\hat{\mu}_\nu < 2.94 \times 10^{-12}(1 + 0.44 H_{13}^2 \rho_6^{-2})^{-1/4} H_{13}^{1/2}. \quad (26)$$

Formula (26) is simplified in two limiting cases, namely relativistic and nonrelativistic ones.

For a nonrelativistic gas ($p_F \ll m$ ($H_{13}/\rho_6 \gg 1$, see Eq. (24)), it takes on the form

$$\hat{\mu}_\nu < 3.61 \times 10^{-12} \rho_6^{1/2}. \quad (27)$$

For a relativistic gas $p_F \gg m$ ($H_{13}/\rho_6 \ll 1$) we obtain

$$\hat{\mu}_\nu < 2.94 \times 10^{-12} H_{13}^{1/2}. \quad (28)$$

The analysis shows that the conditions $p_F \gg m$ and (4) can be simultaneously met only at fairly strong fields H. For example, at $H_{13} = 300$, $\hat{\mu}_\nu < 5.1 \times 10^{-1}$, which is close to bounds (1) and (2). Note that the bound $\hat{\mu}_\nu < 2.9 \times 10^{-11}$, which was found in [7] under the same conditions, should be multiplied by $\sqrt{\pi}$, since it was derived using formula (36) from [6], which has an error in the coefficient: $\pi^{9/2}$ should be replaced by $\pi^{7/2}$ that yields the same bound (28). Such coincidence is explained by the fact that in the relativistic case the luminosity related to the electromagnetic mechanism of the photoproduction $\gamma e \longrightarrow e\nu\bar{\nu}$ (see formula (47) in [7]) is equal to the luminosity (17).

CONCLUSIONS

It was shown in [6] that the plasmon decay plays a significant role in the cooling of strongly magnetized neutron stars and is the predominant mechanism of their energy losses in a broad parameter range. Relative bounds for the effective magnetic moment of the neutrino (namely, formulas (23) and (25)–(28)) reveal the range of its values where the weak channel of the plasmon decay is more effective than the electromagnetic one. In conclusion, we note that production of a neutrino pair by a high-energy photon was considered in [19]. As opposed to the plasmon decay discussed above, this process is caused by coherent interaction of a neutrino possessing a magnetic moment with a dense medium.

ACKNOWLEDGMENTS

The authors are grateful to leading researcher assistant A.E. Lobanov for fruitful discussion of the results.

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Translated by L. Borodina