The theoretical analysis of electronic thermal properties of the interfaces between multiband superconductors and a normal metal

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ABSTRACT

Investigations of the electronic thermal properties of the interfaces between a normal metal and high-temperature superconductors are important for correct design of modern low-temperature electronic refrigerators and bolometers. Multiband superconductivity, recently discovered in ferropnictides and in magnesium diboride, is the suitable choice due to isotropic order parameter in it, in contrast with strongly anisotropic d-wave superconductivity in high-T_c cuprates, which is destructive for electronic refrigeration and bolometric applications. Moreover, recent calculations of Andreev spectra and subgup bound states in ferropnictides, taking into account coherent multiband interference effects in $s_{\pm}$ sign-reversal order parameter model, predict possible suppression of Andreev reflection for clean boundaries between ferropnictides and a normal metal. This Andreev reflection suppression can improve electronic refrigerator quality. Up to now there was no calculation of electronic thermal properties of the interfaces between a normal metal and novel multiband superconductors. In this paper we calculate the thermal flux and electronic thermal conductivity of the boundary between normal metal and novel multiband superconductors. In this calculations both s_{++} and $s_{\pm}$ sign-reversal order parameter models for multiband superconductor is used, taking into account coherent multiband interference effect.

Keywords: multiband superconductors, ferropnictides, microrefrigerators, Andreev reflection

1. INTRODUCTION

The interest in multiband superconductivity reappeared in 2001, when superconductivity in MgB_2 was discovered [1], and this material turned out to be two-gap superconductor [2]. The second significant event in this area was the recent discovery of high-T_c superconductivity in ferropnictides [3, 4]. The most intriguing feature of this materials is the existence of experimental results [5-7], which can be interpreted as corresponding to superconductors with usual s-wave pairing, while others experimental results [8-11] can be successfully interpreted only with assumption of existing nodes in the gap structure. To resolve the contradiction, a new kind of order parameter symmetry was proposed: the $s_{\pm}$ symmetry [12-14]. Now the $s_{\pm}$ model is considered as the most favorable model of superconductivity in ferropnictides.

The important problem is an investigation of the electronic properties of microcontacts involving normal metal (N) and multiband superconductor (S). Investigations of the electronic thermal properties of the interfaces between a normal metal and a high-temperature superconductor are important for correct design of modern low-temperature electronic refrigerators and bolometers [15-17]. Multiband superconductivity is the suitable choice due to isotropic order parameter in it, in contrast with strongly anisotropic d-wave superconductivity in high-T_c cuprates, which is destructive for electronic refrigeration and bolometric applications [17].

The electric current in the normal metal-insulator-superconductor (NIS) contacts is accompanied by the heat transfer from the normal metal into the superconductor [15, 17]. This principle can be applied to the refrigeration of electron gas in the normal metal. Due to the energy gap in the superconductor, electrons with higher energies (above the gap) are

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removed from the normal metal more efficiently than those with lower energies. This leads to the cooling of the electron gas in the normal metal. The effect is similar to the well-known Peltier effect in metal-semiconductor contacts. However, at large transparencies values, coherent two-electron tunneling ("Andreev reflection") starts to dominate and suppress the heat flow. This occurs because in the Andreev reflection process electrons with all energies, including those inside the energy gap, are removed from the normal metal. To solve a problem of calculating the optimal refrigerator parameters one needs to know the probabilities of Andreev and normal reflection of the electron incident from the normal metal to the NIS interface [15,17].

To model the NIS contacts we consider is a constriction between normal metal and superconductor with characteristic dimensions that are much smaller than the superconducting coherence length and inelastic scattering length in the electrodes. These conditions allows one to neglect variation of the superconducting order parameter Δ near the constriction and assume that Δ is constant in space up to the NS boundary and is equal to its equilibrium value inside the superconductor. In this case, one can apply the well-known approach of Blonder, Tinkham and Klapwijk (BTK) [18] to calculate the electric and heat current across NS boundary. The problem of calculation of electric current across the boundary between normal metal and multiband superconductor was recently investigated by Golubov et al [19]. They proposed a modification of BTK model taking into account the presence of two energy bands. In the present work, we have reexamined the model [19] and found out some new details. As to s_{++} model, we find its results to be rather plausible, with the exception of some technical details, which, as will be shown, were missing in the paper [19]. The situation is different in the $s_{\pm}$ model, where an unusual behavior of quasiparticle currents have been found, which, in our opinion, can not be correctly explained within the bounds of this model in its present form.

We will start with a review of the model proposed in [19]. Then we will point out some problems, existing in model [19], and improve this model with the aim to remove this problems. Than, we will compare our results with [19] and point out the suppression of Andreev reflection in both models $s_{\pm}$, s_{++} and discuss the influence of this suppression to the heat current. Then we will discuss the unusual features appearing in the model [19]. In the rest part of the work we will focus on the numerical calculations of the heat current at different values of the model parameters.

A NS boundary is modeled in [19] by a one dimensional conductor, whose right half ($\xi > 0$) is a two-band metal (two different states at the same energy near the Fermi level, one with wave vector π and the other with θ), and the left half is a simple metal. At a normal metal (N) – superconductor (S) interface the wave function is taken in the form:

$$\Psi = \Psi_N \theta(-x) + \Psi_S \theta(x) \,,$$

$$\Psi_N = \psi_k \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a\psi_k \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b\psi_{-k} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{1}$$

$$\Psi_S = c\left[\phi_p \begin{pmatrix} u_1 \\ \upsilon_1 e^{-i\varphi_1} \end{pmatrix} + \alpha\phi_q \begin{pmatrix} u_2 \\ \upsilon_2 e^{-i\varphi_2} \end{pmatrix}\right] + d\left[\phi_p \begin{pmatrix} \upsilon_1 \\ u_1 e^{-i\varphi_1} \end{pmatrix} + \alpha\phi_q \begin{pmatrix} \upsilon_2 \\ u_2 e^{-i\varphi_2} \end{pmatrix}\right]. \tag{2}$$

Here $\varphi_{1,2}$ are the phases of the gaps $\Delta_{1,2}$ in both bands, u and υ are the Bogoliubov coefficients for $E > \max\{\Delta_1, \Delta_1\}$: $u_{1,2}^2 = \frac{1}{2}\left(1 + \frac{\sqrt{E^2 - \Delta_{1,2}^2}}{E}\right)$, $\upsilon_{1,2}^2 = \frac{1}{2}\left(1 - \frac{\sqrt{E^2 - \Delta_{1,2}^2}}{E}\right)$, and α is a mixture parameter defining the relative weight of two bands in the total wave function. The authors[19] assume that α can depend on several factors, which are not only the bulk material properties, but also the properties of interface [19]. With this in mind, the authors keep α arbitrary and present the results for different values of this parameter. The phase difference $\varphi_1 - \varphi_2$ distinguishes two models of the extended s-symmetry. For the s_{++} model (which is applied to materials like MgB_2) it is assigned to be $\varphi_1 = \varphi_2$, while the $s_{\pm}$ model (associated with ferropnictides) is characterized by $\varphi_1 - \varphi_2 = \pi$. The amplitudes $\alpha; \beta$ describe Andreev and normal reflection, and the amplitudes $\chi; \delta$ describe transmission without branch crossing and with branch crossing, respectively.

The total wave function satisfies the usual boundary conditions at the interface ($x = 0$):

$$\Psi(0) = \Psi_S(0) = \Psi_N(0)\,, \quad (3)$$

$$\frac{\hbar^2}{2m}\frac{d}{dx}\Psi_S(0) - \frac{\hbar^2}{2m}\frac{d}{dx}\Psi_N(0) = H\Psi(0)\,, \quad (4)$$

where H is the strength of the (specular) barrier. Introducing the dimensionless barrier strength $Z = \frac{H}{\hbar \upsilon_N}$, where υ_N is the Fermi velocity on the N side of the contact (see [19] for details), and applying the ansatz (1)-(2) to the boundary conditions (3)-(4), one can find the general solution for the coefficients $\alpha; \beta; \chi; \delta$ in the same manner as in usual BTK model [18]. This solution depends on Z and also on the ratios of the Fermi velocities in normal metal and both bands of superconductor. In paper [19] the results were presented for the equal Fermi velocities on the N side and in both bands of the S side. In the present paper, we accept this assumptions. The results are summarized in Table 1. In a single band case (α = 0) the standard BTK results are recovered.

$S_\pm$ model		S_{++} model	
$a = \left(u_1\upsilon_1 - \alpha(u_1\upsilon_2 + u_2\upsilon_1) + \alpha^2 u_2\upsilon_2\right)/\gamma$ $b = \left((Z^2 - iZ)\left[\upsilon_1^2 - u_1^2 + \alpha^2(u_2^2 - \upsilon_2^2)\right]\right)/\gamma$ $c = \left((1 - iZ)(u_1 - \alpha u_2)\right)/\gamma$ $c = \left(iZ(\upsilon_1 - \alpha\upsilon_2)\right)/\gamma$ $\gamma = (1 + Z^2)(u_1^2 - \alpha^2 u_2^2) - Z^2(\upsilon_1^2 - \alpha^2\upsilon_2^2)$	(5)	$a = \left(u_1\upsilon_1 + \alpha(u_1\upsilon_2 + u_2\upsilon_1) + \alpha^2 u_2\upsilon_2\right)/\gamma$ $b = \left((Z^2 + iZ)\left[(\upsilon_1 + \alpha\upsilon_2)^2 - (u_1 + \alpha u_2)^2\right]\right)/\gamma$ $c = \left((1 - iZ)(u_1 + \alpha u_2)\right)/\gamma$ $c = \left(iZ(\upsilon_1 + \alpha\upsilon_2)\right)/\gamma$ $\gamma = (1 + Z^2)(u_1 + \alpha u_2)^2 - Z^2(\upsilon_1 + \alpha\upsilon_2)^2$.	(6)

Table 1. Amplitudes of the waves in BTK model for the cases of $s_\pm$ and s_{++} models.

Now, the probabilities of Andreev and normal reflections can be defined, as usually, as $A = |a|^2$ and $B = |b|^2$ respectively. To compute the probabilities C and D for transmission without branch crossing and with branch crossing, respectively, one can use the expression for probability current [18]:

$$J_P = \frac{\hbar}{m}\left[\mathrm{Im}(f^*\nabla f) - \mathrm{Im}(g^*\nabla g)\right], \quad (7)$$

Equation (7) can be presented in the form

$$J_P = \frac{\hbar k}{m}(C + D)\,, \quad (8)$$

where probabilities C and D can be found by setting the wave functions (2) to (7). We carried out this computation and have got the following results:

$$C = |c|^2 \,\mathrm{sign}E\left((|u_1|^2 - |\upsilon_1|^2) + \alpha^2(|u_2|^2 - |\upsilon_2|^2) + 2\alpha\,\mathrm{Re}(u_1 u_2^* \pm \upsilon_1\upsilon_2^*)\right), \quad (9)$$

$$D = |d|^2 \,\mathrm{sign}E\left((|u_1|^2 - |\upsilon_1|^2) + \alpha^2(|u_2|^2 - |\upsilon_2|^2) - 2\alpha\,\mathrm{Re}(\pm u_1 u_2^* + \upsilon_1\upsilon_2^*)\right), \quad (10)$$

where the upper sign correspond to the $s_\pm$ model, and the lower sign correspond to the s_{++} model. It is necessary to note that our relation for probability D differ from the same formula in the original paper [19]. From our point of view, it is an obvious misprint in [19]. For example, one can see that the condition of probability current conservation A + B + C + D = 1 [18] does not hold true with C and D taken as in [19], while it holds true with our expressions (9)-(10) with some nuances discussed below.

Using the probabilities A, B, C and D, one can write the relations for electron distributions moving to and from the NS interface [18,15]. Energy distribution of electrons $f_\rightarrow(E)$ that move from the bulk of the normal metal to the NS interface is the equilibrium Fermi distribution shifted by eV: $f_\rightarrow(E) = f_F(E - eV)$. Electrons moving from the interface into the normal metal are produced in three processes [18,15]:

(a) quasiparticles incident from the superconductor are transmitted into the normal metal with the probability $1 - A - B$;
(b) electrons are reflected from the interface with probability B;
(c) holes are Andreev reflected as electrons with probability A.

Thus, the energy distribution $f_{\leftarrow}(E)$ of electrons moving into the normal metal is

$$f_{\leftarrow}(E) = A(E)\left[1 - f_{\rightarrow}(-E)\right] + B(E) f_{\rightarrow}(E) + \left[1 - A(E) - B(E)\right] f(E) \quad . \tag{11}$$

The relation for the heat current flowing from the normal metal into the superconductor in one (spin-degenerate) transverse mode has the form [15]:

$$j = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} dE(E - eV)\left[f_{\rightarrow}(E) - f_{\leftarrow}(E)\right]. \tag{12}$$

This relation has the similar form as well known relation for electron current in one (spin-degenerate) transverse mode [18]:

$$J = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} dE\left[f_{\rightarrow}(E) - f_{\leftarrow}(E)\right]. \tag{13}$$

The last step in the calculation is a summation over transverse modes. In paper [15] this summation is done by multiplying (12,13) by the effective number *N* of transverse modes per unit square of the interface.

2. THE BOGOLIUBOV COEFFICIENTS AT SUBGAP REGION

Now we point out some important details which were missing in the original paper [19]. The authors pay attention to the new effects arising in the $s_{\pm}$ model: unusual suppression of Andreev reflection and subgap bound states. For the $s_{\pm}$ case in the transparent interface Z = 0 they have obtained: $b = d = 0$, $a = (\upsilon_1 - \alpha\upsilon_2)/(u_1 + \alpha u_2)$, $c = 1/(u_1 + \alpha u_2)$. At zero energy $E = 0$ they have obtained $a = (\sqrt{\Delta_1} - \alpha\sqrt{\Delta_2})/(\sqrt{\Delta_1} + \alpha\sqrt{\Delta_2}) < 1$, i.e. Andreev reflection is suppressed. It means that some ratio of probability of incident electron tunnel through a clean interface at subgap region of energies, which do not take place in case of conventional superconductors. As this result is essentially unusual, we were interested of how it was obtained in details. We recognized that is the question how to continue the Bogoliubov coefficients $u_{1,2}$ and $\upsilon_{1,2}$ to subgap region of energies. To the best of our knowledge, these continuations are not usually given in explicit form. We have derived these continuations using some natural physical conditions. Using these continuation of Bogoliubov coefficients we have got the results different from [19] in subgap and between-gap regions.

We define the "subgap region" the values of energies $|E| \le \min(\Delta_1, \Delta_2)$ and "between-gap region" the values of energies satisfying the condition $\min(\Delta_1, \Delta_2) < |E| < \max(\Delta_1, \Delta_2)$. We have got the continuations of u_i and υ_i (i = 1, 2) to the regions $|E| < \Delta_i$ using the following natural physical conditions:

(a) the wave transmitted to superconductor from normal metal must decay when $|E| < \Delta_i$;

(b) the normalization condition has the form $|u_i|^2 + |v_i|^2 = 1$;

(c) the coefficients u_i and υ_i are continuous functions of *E* at $|E| = \Delta_i$ and satisfy to Bogoliubov equations [18].

From conditions (a)-(c) follows the relations for u_i, υ_i at the region $|E| < \Delta_i$:

$$u_i = \frac{1}{2}\left(\sqrt{1 + \frac{|E|}{\Delta_i}} + i\sqrt{1 - \frac{|E|}{\Delta_i}}\right)\mathrm{sign}E \;; \qquad \upsilon_i = \frac{1}{2}\left(\sqrt{1 + \frac{|E|}{\Delta_i}} - i\sqrt{1 - \frac{|E|}{\Delta_i}}\right). \tag{14}$$

With this in hand, we have verified the equality A + B + C + D = 1 and recognize, that it is true for all energies *E* only when our correct expressions (9), (10) for coefficients C, D are used. Then we have re considered the effects arising at subgap and between-gap regions in the $s_{\pm}$ model again. For E → 0 for Z = 0 we obtained:

$$A = \left(\frac{1-\alpha}{1+\alpha}\right)^2; \qquad C = \frac{4\alpha}{(1+\alpha)^2}; \qquad B = D = 0 \tag{15}$$

One can see that results (15) differ from those at [19], however, they have resembling qualitative features. Andreev reflection is suppressed (A < 1), and the transition coefficient C is not zero, i.e. some ratio of incident electrons tunnel through a clean interface at subgap region (see Fig. 1). The picture is most unusual when α = 1: there is no Andreev reflection at zero energy. Formally, A + C = 1 (probability current is conserved), but the physical interpretation of this fact is not quite clear: we have deal either with a new non-trivial effect, or with a qualitative disadvantage of this model for $s_{\pm}$ superconductors.

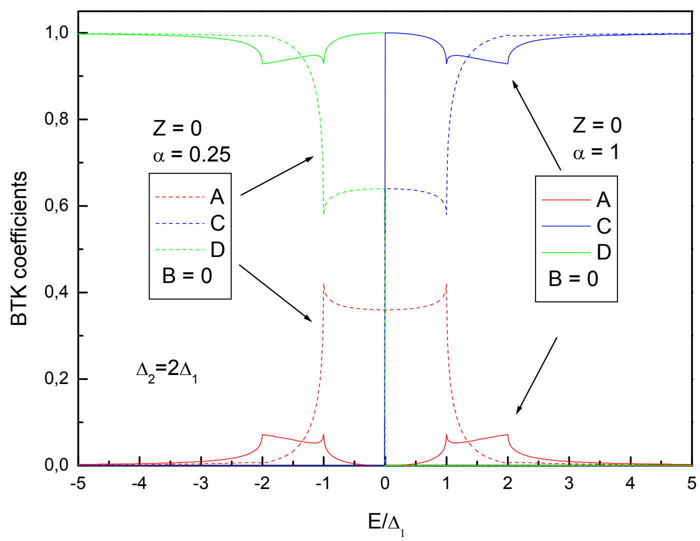

Fig. 2. BTK coefficients versus normalized energy, the case of $\Delta_2 = 2\Delta_1$ and transparent interface, $s_{\pm}$ model. Andreev reflection is suppressed while transmission coefficients are not zero at subgap region. One can also notice the unusual sudden change of C and D at E = 0. The last does not take place in s_{++} model, where C = D = 0 at subgap region.

3. THE SUPPRESSION OF ANDREEV REFLECTION AND ITS INFLUENCE TO THE HEAT CURRENT

In the s_{++} model the behavior of BTK coefficients *A*, *B*, *C*, *D* is much more clear, and we will focus now on this case. Here there is a resembling effect of suppression of Andreev reflection, but it differs from $s_{\pm}$ model (see Fig. 2). In s_{++} model, the Andreev suppression takes place only in case of finite Z (non-transparent interface).

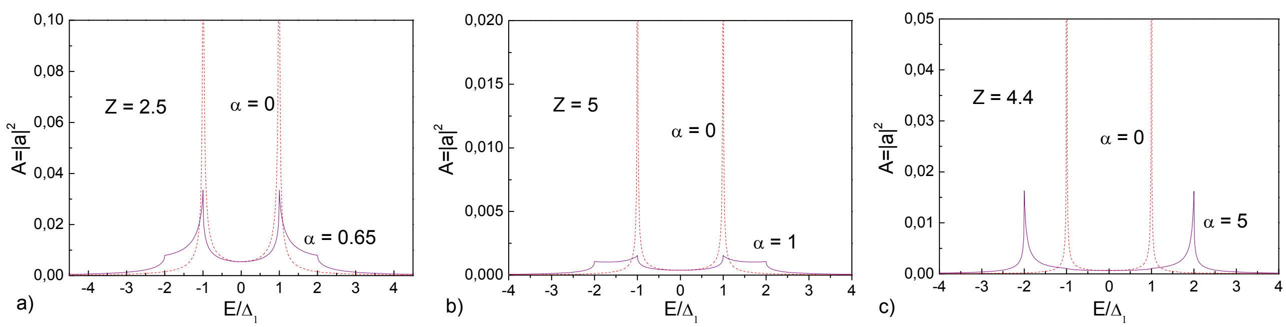

Fig. 2. Andreev reflection versus normalized energy for $\Delta_2 = 2\Delta_1$, s_{++} model. Comparison of conventional superconductor (case α = 0) and two-band superconductor. a) High transparency, medium mixture parameter. The unity-height peaks at the smaller gap decrease drastically. b) Medium transparency, medium mixture parameter. Andreev reflection is reduced to low plateaus at subgap and between-gap regions. c) Medium transparency, high mixture parameter. Andreev reflection arises at the larger gap.

It is clear qualitatively that the suppression of Andreev reflection enhances the cooling power of the normal metal, as it prevent the electrons with low energies (including those inside the smaller gap) to leave the normal metal. At the between-gap region of energies an additional Andreev reflection appears due to “switching” the larger gap on. This do not prevent the cooling of the normal metal, because this region lies at higher energies and additional Andreev reflection removes just more “hot” electrons. The increasing of the relative weight of the larger gap (i.e. the increasing of α) the region of intensive Andreev reflection moves towards higher energies and the heat flow from the normal metal increases. However, with increasing of α, the weight of the smaller gap decreases, and one can expect that the heat flow will diminish. The last takes place because the smaller gap helps to keep the “cool” electrons inside the normal metal. We come to conclusion that the heat current from the normal metal can reach its maximum at the concrete value of α. The numerical calculations confirm this conclusion. If there is no smaller gap at all (the case of a usual superconductor with a single gap) the heat current is less than in case of optimal α. In other words, existence of two gaps provides a winning in the cooling power.

Note that in case of $s_{\pm}$ superconductors the suppression of Andreev reflection does not enhance the heat current so, as in s_{++} case. The numerical calculations (see section 5) shows that in the case of $s_{\pm}$ model the winning in the cooling power is smaller than in case of s_{++} model.

4. THE HARDSHIPS OF THE S± MODEL

Now we pay attention to another drawback in the $s_{\pm}$ model of the interface. One can easily see that BTK coefficients A, B, C, D defined by (5), (9) and (10) diverge (have singularities) under some values of α and Z parameters.
Also, one can see that D may become negative.

Moreover, we have shown that coefficient D always has negative or zero values within subgap and between-gap regions. This strange behavior of C and D coefficients is summarized on Fig. 3. There are three kinds of singularities.

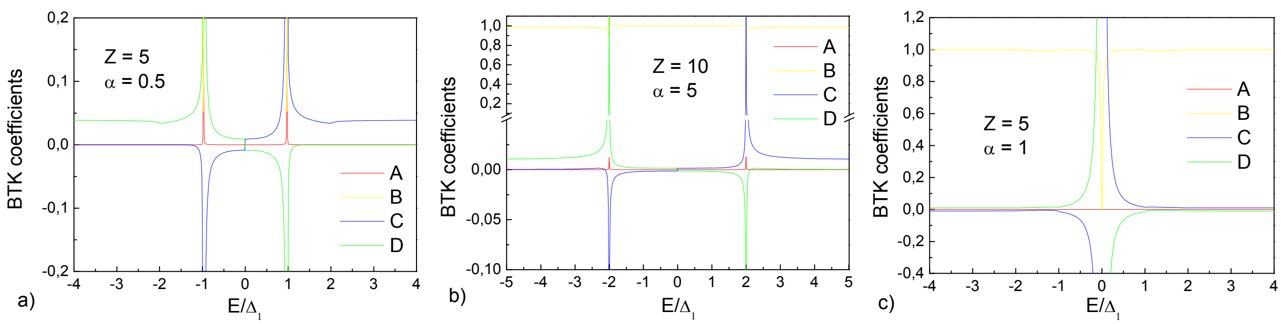

Fig. 3. The behavior of BTK coefficients in case of $s_{\pm}$ model, $\Delta_2 = 2\Delta_1$. a) The singularity of type-1 occurs near the smaller gap in case of low and medium mixture parameter α except to the special case α = 1. b) The singularity of type-2 takes place near the larger gap (above the gap) under sufficiently high mixture parameter. The estimate of such α is given below. c) The singularity of type-3 occurs at zero energy only at the special case α = 1 when Z > 0.

We have to stress now, that the demonstrated behavior of BTK coefficients (singularities and negative values) has no clear physical interpretation with the framework of the model [19]. In paper [19], the singularities were associated with Andreev bound states. The negative values of D (or C when $E < 0$) formally means that the currents of quasiparticle excitations in superconductor flowing towards the normal metal associated with an electron incident from the normal metal, what is not clear at all.

We have tested numerically the possibility of calculating the electric (13) and heat currents (12) across the interface for several values of parameters of model [19]. It was found that current integrals (12,13) converge in cases of singularities of type-1 and type-3, and diverge in case of singularity of type-2. Therefore, if one calculates the electric or heat current, he must avoid the singularities of type-2 (otherwise the result will diverge). We have found the conditions necessary to keep $D \geq 0$ and $C \geq 0$ in the region $|E| \geq \max(\Delta_1, \Delta_2)$ for arbitrary Z in the following form:

$$\alpha \geq \alpha_{\min} = \sqrt{\frac{2\Delta_1}{\Delta_1 - \Delta_2}} \quad \text{in case } \Delta_1 > \Delta_2, \tag{16a}$$

$$\alpha \leq \alpha_{\max} = \sqrt{\frac{\Delta_2 - \Delta_1}{2\Delta_2}} \quad \text{in case } \Delta_2 > \Delta_1. \tag{16b}$$

Note that for $\Delta_1 = \Delta_2$ and Z > 0 we always have negative values of D above the larger gap. Conditions (16a) and (16b) ensures $D \geq 0$ and $C \geq 0$ in a small vicinity of the larger gap. Strictly speaking, they are not sufficient conditions for keeping C and D positive. Taking into account that singularity of type-2 appears not far from the larger gap, we consider (16a) and (16b) as an estimation of range of α, at which calculations of electric and heat currents are possible. For the case $\Delta_2 = 2\Delta_1$, considered on Fig. 3, it gives $\alpha \le \alpha_{\max} = 1/2$.

Keeping in mind our remarks about restriction (16a),(16b) for the two band BTK model [19] and our results for Bogoliubov coefficients (14) in subgap and between-gap regions and revised expressions for C and D (9),(10) we have calculated the conductance dI/dV for zero temperature from formula (13) (Fig. 4). The set of parameters is taken the same as in the work [19], and our results, as expected, differ from those in [19] at subgap and between-gap regions.

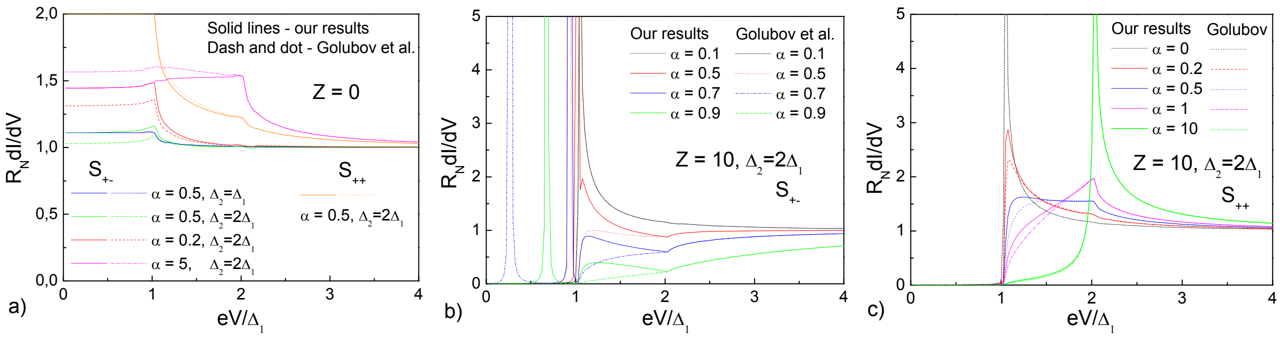

Fig. 4. Conductance. Comparison of our results and Golubov et al. [19] a) Fully transparent interface, Z = 0. Comparison of the s_{++} and $s_{\pm}$ models. b) Conductance in the low transparency regime, Z = 10, in the $s_{\pm}$ model. c) Conductance in the low transparency regime, Z = 10, in the s_{++} model.

At Fig. 4b only the first two pairs of curves correspond to the values of α satisfying (16b). In the other cases (exactly α = 0.7 and α = 0.9; Z = 10) the condition (16b) is violated, however, the calculation of current and conductance is formally possible.

One may note that if conditions (16) are violated, then our results differ from results [19] qualitatively; in particular, [19] predicts sharp conductance peaks at subgap region even if (16) are fullfilled, while in our picture such peaks appear at subgap region only when conditions (16) are violated. In other cases the peaks appear at the smaller gap or at between-gap region (likewise in the usual BTK model). It is worth noting that, in spite of these conductance peaks is associated with Andreev bound states in [19], they are not related with them directly. Perhaps, the peaks at subgap region might be just artifacts of the model [19].

We also must point out one additional problem of the $s_{\pm}$ model of the interface [19]. We have calculated the probabilities C' and D' for the quasiparticle incident from the superconductor to the normal metal to traverse across the interface as an electron and as a hole respectively. We have demonstrated, that the equalities $C'N_s = C$, $D'N_s = D$, where N_s is density of states of quasiparticle excitations in superconductor, cannot be satisfied simultaneously in case of $s_{\pm}$ model. These equalities express the condition of detailed equilibrium of quasiparticle currents and are satisfied in the usual BTK model [18]. However, we have found that $\frac{C}{C'} \ne \frac{D}{D'}$, so, the last conditions can not fulfilled simultaneously. One may assume that the condition of equilibrium of NS boundary has the form $(C'+D')N_s = C + D$ and find N_s from this relation. But in this case N_s turns out to be dependent on barrier strength Z, that has no physical treatment within the model [19].

5. THE HEAT CURRENT

In this chapter we present our results of numerical calculations of the heat current at different values of parameters of s_{++} and $s_{\pm}$ models, that are the temperature of the normal metal T, bias voltage V, transparency of the interface D (or barrier strength Z), and the mixture parameter α. One can also consider different values of energy gaps Δ_1 and Δ_2, at this case the additional parameter is their ratio Δ_2/Δ_1. Our purpose is to find the optimal regime of cooling of normal metal.

We have chosen the ratio of energy gaps as $\Delta_2 = 2\Delta_1$, as in [19]. It was found in paper [15], that in the case of usual one-band s-wave superconductors the heat current reaches its maximum at the temperature $T \approx 0.3\Delta$. Now we consider two-band superconductor with $\Delta_2 = 2\Delta_1$, and one can expect that the heat current will have a maximum at some optimal temperature $T_{opt} \in [0.3\Delta_1; 0.6\Delta_1 = 0.3\Delta_2]$. Taking into account the suppression of Andreev reflection discussed in section 2, one can also expect that the optimal transparency of the interface will be shifted towards the higher values. However, manufacturing of boundaries with high transparency is still concerned with technical difficulties [16]. Taking into account these factors, we will start with fixed temperature $T = 0.3\Delta_1$ and with intermediate value of barrier strength Z = 10. Figure 5 shows the heat current as a function of the bias voltage V across the contact for fixed T and Z and different values of α. We see that for each α there is an optimal bias voltage which maximizes the heat current, and we can also evaluate the optimal value of α, which is different for s_{++} and $s_{\pm}$ model. In Fig. 5 and in following figures we normalized the heat current on factors, including the value of smaller gap Δ_1 only.

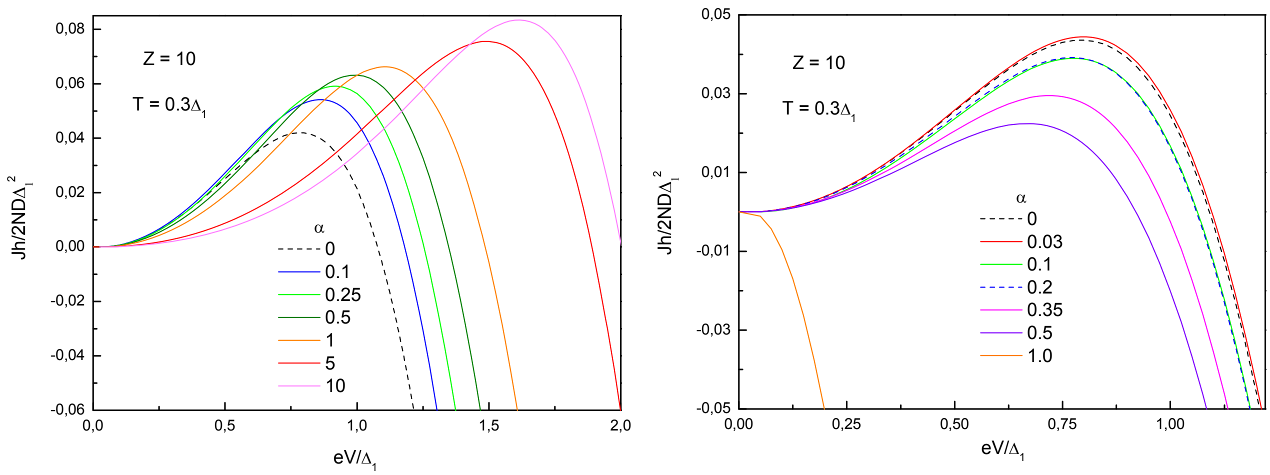

Fig. 5. Normalized heat current versus bias voltage. Left: s_{++} model. The heat current reaches maximal values at α ~ 5. Right: $s_{\pm}$ model. The maximal values of heat current correspond to small α ~ 0.03. The behavior of heat current becomes unusual when the conditions (16) are violated.

We see that in the case of s_{++} model the heat current increases with increasing of α in accordance with qualitative arguments given in section 3. Comparing the case α = 5 with the usual single-band superconductor (α = 0) we see that existence of the second gap provides a "winning" in cooling power by the value ~0.75/0.4 ≈ 1.88. However, one must take into account that heat current is normalized on the smaller gap, and, with increasing α, the larger gap increases its relative weight, therefore the heat current increases. If we normalize the heat current to the larger gap, the results will decrease by 4 times (squared ratio of the gaps). So, the "pure winning" is just about 1.88/4 ≈ 0.47, i.e. there is no winning at given set of the parameters. In case of $s_{\pm}$ model the (visible) winning is very small and it is achieved at small value of α ~ 0.03. On the other hand, the parameters $T = 0.3\Delta_1$, Z = 10 are possibly far from optimal regime. If we consider α ~ 5, the optimal temperature is somewhat about $0.3\Delta_2 = 0.6\Delta_1$. At next step we will search for the optimal temperature. We also note that the values α ~ 5 and higher may turn out to be not available physically. We consider the case α ~ 2 as possibly useful for applications. As to $s_{\pm}$ model, there is an additional restrictions (16) on α. The behavior of heat current becomes strange when conditions (16) are violated.

As bias voltage can usually be easily tuned in the experiment, we shall not focus on the concrete values of optimum bias voltage and present the results for optimum bias voltage for the set of other parameters. Figure 6 shows the heat current as a function of temperature for the optimal bias voltage.

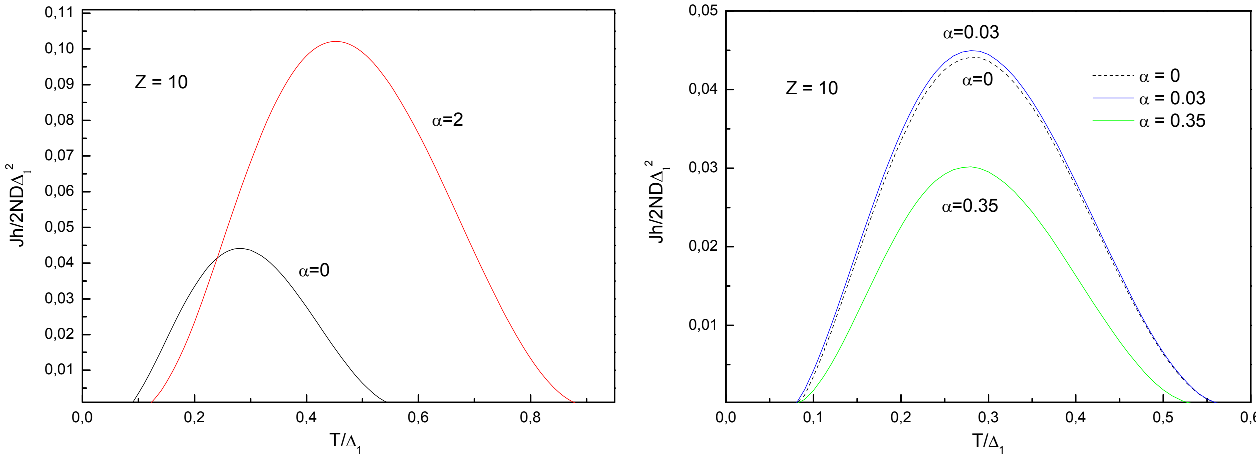

Fig. 6. Normalized heat current calculated for the optimum bias voltage as a function of temperature. Left: s_{++} model. The heat current at α = 2 takes on its maximum value at $T \approx 0.45\Delta_1$. Right: $s_\pm$ model. The optimal temperature is near to the magnitude $0.3\Delta_1$ due to small values of α.

As was expected, in case of s_{++} model with α = 2 the heat current reaches its maximum at $T \approx 0.45\Delta_1$. The visible winning in cooling power is at about 2.4 times. The "pure winning" is still less than 1, i.e. there is no real winning in cooling power. In case of $s_\pm$ model the optimal temperature is about $0.3\Delta_1$ and even the visible winning is small.

In Fig. 7 we plot the heat current at the optimum bias voltage as a function of barrier transparency D. At small transparencies the heat current increases linearly with D, because in the tunneling limit the electron transport is dominated by single-particle tunneling. At larger transparencies the heat current starts to decrease with D due to the increasing contribution to transport from two-particle tunneling (Andreev reflection) [15].

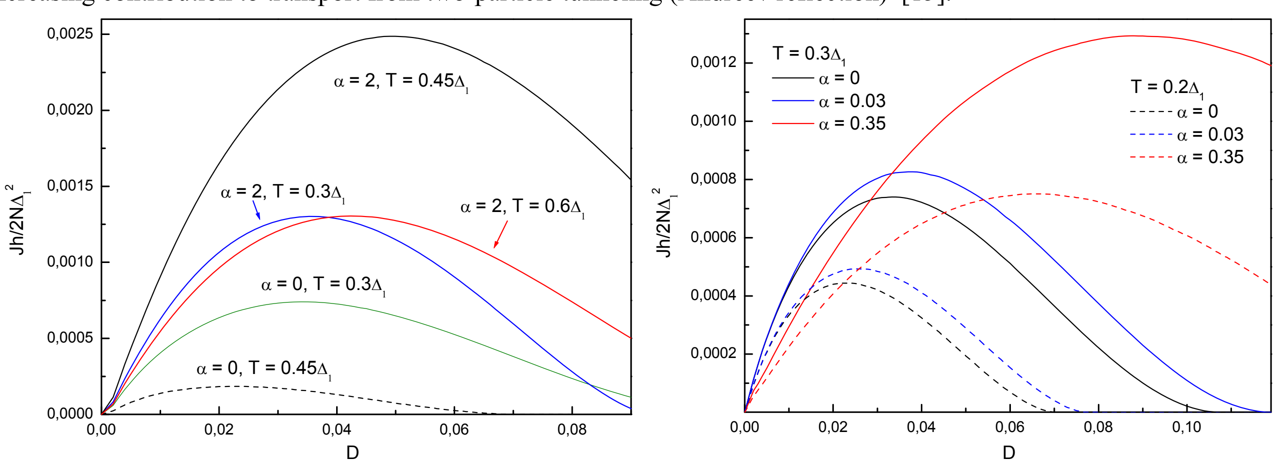

Fig. 7. Normalized heat current for optimal bias voltage versus transparency D of the interface. Left: s_{++} model. The optimal regime with fixed α = 2 is achieved at $T \approx 0.45\Delta_1$ and D = 0.049. Right: $s_\pm$ model. Two optimal regimes can be marked out: α = 0.35, $T \approx 0.3\Delta_1$, D = 0.09 and α = 0.03, $T \approx 0.3\Delta_1$, D = 0.037 .

As was mentioned before, the optimum transparency moves towards higher values with increasing α due to the suppression of Andreev reflection in model [19]. In the case of s_{++} model with α = 2 the optimal regime is achieved at $T \approx 0.45\Delta_1$ and $D = 0.049$, that corresponds to Z = 4.4. The winning in cooling power, compare to a single-band superconductor (α = 0, $T = 0.3\Delta_1$), is about $\eta = 3.6$ times. The lower bound of "pure winning" is $\eta \big/ ((1/1+\alpha^2) + (\alpha^2/1+\alpha^2)\cdot(\Delta_2/\Delta_1)^2) = 1.06 > 1$. In the case of $s_\pm$ model there are two good regimes: α = 0.35, $T \approx 0.3\Delta_1$, D = 0.09 (that corresponds to Z = 3.18) and α = 0.03, $T \approx 0.3\Delta_1$, D = 0.037 (Z = 5.1). The former provides the greater heat current, but requires high transparency of the interface. The "pure winning" compare to a single band superconductor is less than unity in both regimes for $s_{\pm}$ model.

In the end we plot the heat current as a function of mixture parameter α taking into account the values of parameters that were found to be close to the optimal regime (Fig. 8).

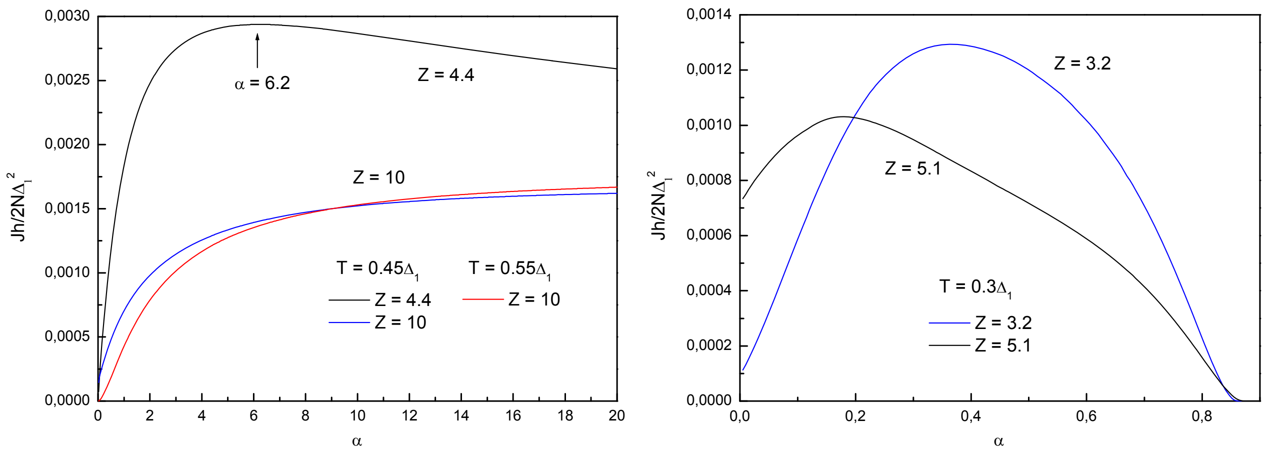

Fig. 8. Normalized heat current (optimized by bias voltage) versus mixture parameter α. Left: s_{++} model. The optimal regime is achieved at α = 6.2, $T \approx 0.45\Delta_1$ and D = 0.049. The regimes with lower transparency are given for comparison. Right: $s_{\pm}$ model. Two optimal regimes are detected: α = 0.37, $T \approx 0.3\Delta_1$, D = 0.09 (high transparency) and α = 0.18, $T \approx 0.3\Delta_1$, D = 0.037 (medium transparency).

We see that in the case of s_{++} model the optimal regime is achieved at α = 6.2, $T \approx 0.45\Delta_1$ and D = 0.049 (Z = 4.4). The value of normalized heat current at α = 0 and the transparency optimal for this case can be evaluated from Fig. 4: $J(T = 0.3\Delta_1) \approx 7.3 \cdot 10^{-4}$. Thus, the "pure winning" in cooling power in the optimal regime can be evaluated (taking into account $\alpha_{opt}^2 >> 1$) as follows: $J_{opt}(\alpha = 0)/4J_{opt}(\alpha_{opt}) = 0.99 < 1$, i.e. the optimal regime does not provide winning in cooling power. The largest "pure winning" we have detected is approximately 6% at α = 2, $T \approx 0.45\Delta_1$ and D = 0.049 (that corresponds to Z = 4.4). The "pure winning" in case of $s_{\pm}$ model is even smaller.

6. CONCLUSIONS

We have demonstrated the necessity of continuation of the Bogoliubov coefficients under the gaps and have obtained the correct expressions for the BTK coefficients C and D. This allowed us to display the effect of suppression of Andreev reflection in both s_{++} and $s_{\pm}$ models and ascertain that it takes place in the two models in qualitatively different ways. As to s_{++} model, we regard it as a correct approximation in a known sense. At the same time, the proposed extension of BTK model for $s_{\pm}$ superconductors leads to non-usual results as discussed above and seems to have fundamental hardship.

We have considered the behavior of heat current as a function of temperature T, bias voltage V, transparency of the interface D and the mixture parameter α. It has been found that the suppression of Andreev reflection enhances the heat current from the normal metal and shifts the optimal transparency of the interface towards the higher values. This effect takes place in both s_{++} and $s_{\pm}$ models, but the enhancement of heat current is greater in the case of s_{++} model. However, the "pure winning", compare to a single band superconductor with larger gap, in cooling power is insignificant. On the other hand, the recently discovered multiband superconductors can be used in electronic refrigerators due to the large values of their energy gaps that can provide the greater cooling power than single-band superconductors used before, and due to isotropic order parameter in it, in contrast with anisotropic d-wave superconductivity in high-T_c cuprates, which suppresses the cooling effect strongly.

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