Violation of dispersion relations in magnetotellurics

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Introduction

Kramers-Kronig (Bode) dispersion relations connect the real and imaginary parts (amplitude and phase) of physically realizable transfer functions [Nussenzweig, 1972]. These integral relations are the direct consequence of the principle of causality and are widely used in many branches of Physics (including EM Geophysics) to check the consistency of measured data and increase their quality.

It is important to mark the distinction between the "real-imaginary" dispersion relation of the first kind (**DR-I**), and the "amplitude-phase" dispersion relation of the second kind (**DR-II**), which for the components of a causal spectrum $F(\omega)$ could be written as follows:

$$Im\{F(\omega)\} = \frac{2\omega}{\pi} p.v. \int_0^{+\infty} \frac{Re\{F(\omega)\}}{\omega_0^2 - \omega^2} d\omega_0 \qquad \textbf{(DR-I)}$$

$$arg\{F(\omega)\} = \frac{2\omega}{\pi} p.v. \int_0^{+\infty} \frac{\ln|F(\omega)|}{\omega_0^2 - \omega^2} d\omega_0 \qquad \textbf{(DR-II)}$$

The principal difference between these two relations is that the **DR-I** is valid in any causal transfer function, while the **DR-II** holds only for a special class of causal functions which are called minimum-phase.

If the black box system under consideration is assumed to be linear, time-invariant and passive (Fig. 1), any input-output transfer function must be causal, so the **DR-I** always holds in controlled-source methods of EM geophysics. From the other hand, the output-output transfer functions employed in magnetotelluric (MT) exploration connect various components of electric and magnetic field on the Earth surface induced by remote natural sources (lightnings, ionospheric/magnetospheric currents, etc.) and thus are not necessarily causal: the causality of MT impedance function is rigorously proved only for 1D [Weidelt, 1972] and some types of 2D [Weidelt, Kaikkonen, 1994] models. As a result, the existence of the dispersion relations in 3D models and real geological conditions turned out to be one of the most controversial subjects of modern magnetotellurics [Berdichevsky, Dmitriev, 2008; see also Egbert, 1990; Yee, Paulson; 1990, etc.].

In the recent decade in the MT community there were proposed several synthetic models and reported numerous reliable field observations showing dramatic violation of **DR-II** between the spectral components of MT impedance tensor ("phase rolling out of quadrant"), which seems to be a conclusive evidence for existence (and, apparently, rather wide occurrence) of non-minimum-phase magnetotelluric responses. However, most of these reports never considered the possible validity of **DR-I**, so the question about the *causality* of the measured transfer functions remained open. As still remain open other key questions on the subject, such as:

- **Do non-causal MT response functions actually exist, and, if so, in which situations and geological conditions could they be encountered?**
- **Is it possible to tell the difference between the dispersion relations' violation caused by inconsistent data and that caused by geological reasons?**
- **Can we somehow use the dispersion relations to increase the quality of measured data for non-minimum-phase and non-causal transfer functions?**
- **What is the best strategy for inversion of such data?**

etc.

Research summary

Trying to answer the above questions we have come to a theoretical prediction that under the conventional black box assumptions (Fig. 1) there could generally be encountered only **four types** of MT transfer functions, each with its own set of rules for the dispersion relations to follow [Zorin, Alekseev, 2018].

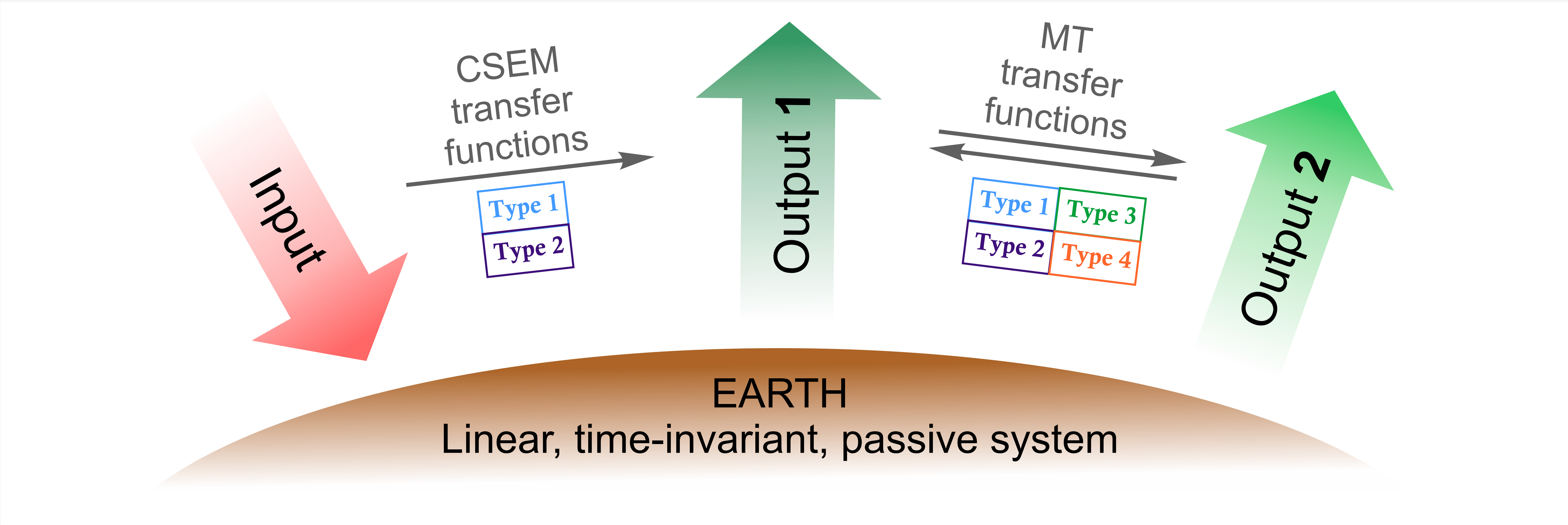
In the present work we partially verify the validity of this prediction on the MT impedance curves.

Discussion and Further Research

We have successfully found several synthetic and field examples for the MT curves of the types 1–3 (including the first appropriately confirmed illustration of a non-causal impedance in MT literature), however a reliable example of the 4-th type curve is yet to be discovered.

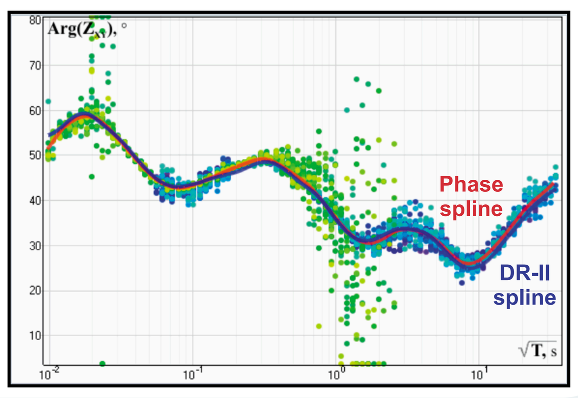
Another thing necessary for acknowledgment of the proposed theory is empirical confirmation that there are no other types of **DR-II** violation except of described above. For this purpose we are going to collect and examine an extensive statistics of "anomalous" synthetic/field data, and will appreciate any corresponding contribution from the MT community.

Fig. 1. Controlled-source and natural-source EM measurements on a "black box" Earth model scheme



Type 1

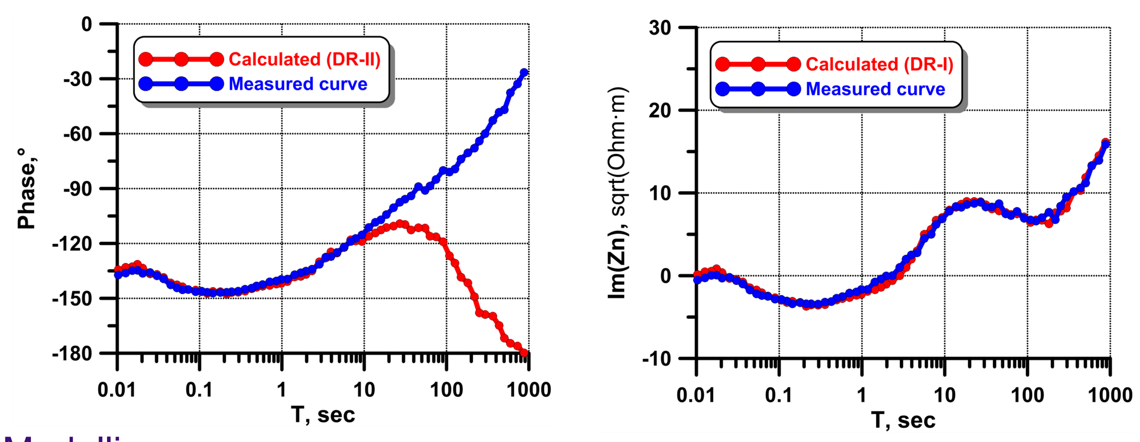
Causality: **causal, minimum-phase**
Validity of dispersion relations: **DR-I, DR-II**
Abundance for MT impedance components: **most common**
Field data example (Nord-West Ltd.):



Modelling:
This type of data is observed for MT impedance functions calculated on the surface of all isotropic 1D/2D models and most part of simple 3D models.

Type 2

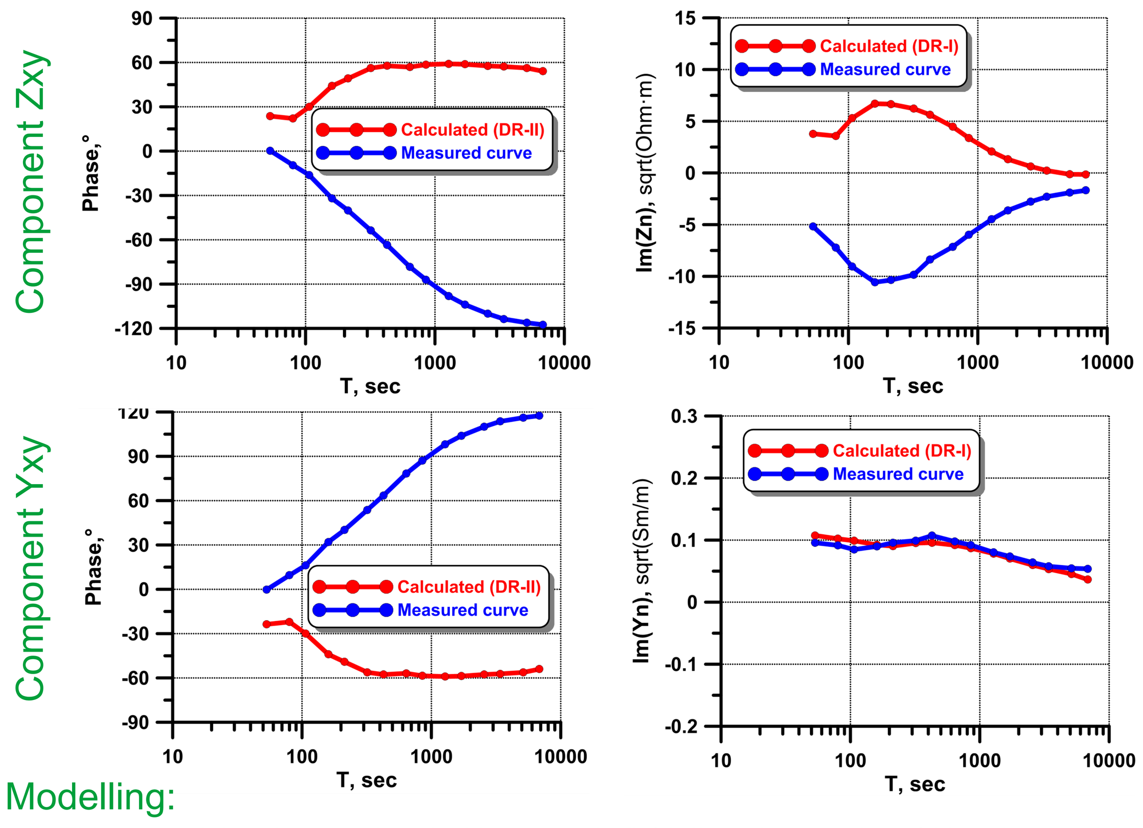
Causality: **causal, non-minimum-phase**
Validity of dispersion relations: **only DR-I**
Abundance for MT impedance components: **common**
Field data example [Ichihara et al., 2013]:



Modelling:
This type of data is associated with electric field distortions and is often observed over the bends and edges of highly conducting objects in some 3D models [eg. Ichihara, Mogi, 2009; Kaufman et al., 2014] and (for Zxx and Zyy components) in anisotropic media [Marti, 2014].

Type 3

Causality: **non-causal, with causal reciprocal**
Validity of dispersion relations: **none**
Abundance for MT impedance components: **rare**
Field data example [Baba et al., 2017]:



Modelling:
This type of data is associated with magnetic field distortions and could be observed for the seafloor measurements in the presence of strong coastal effect [eg. Alekseev et al., 2009; Kapinos, Brasse, 2011].

Type 4

Causality: **non-causal, with non-causal reciprocal**
Validity of dispersion relations: **none**
Abundance for MT impedance components: **exceptional**
Field data example: (to be found)
Modelling: (no examples yet)

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