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$Z \to e^+e^-$ DECAY IN A CONSTANT MAGNETIC FIELD

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In the one-loop approximation we calculate the electronic contribution to the elastic scattering amplitude of a Z-boson in a constant homogeneous magnetic field. Using this amplitude we obtain the probability of the decay of a Z-boson into an e^+e^- pair, and we investigate its dependence on the boson energy and the external field strength.

The Weinberg–Salam–Glashow theory of electroweak interactions agrees well with experiment, and the existence of the intermediate vector bosons, the carriers of the weak interactions [1], has been demonstrated experimentally. Therefore, it is of interest to study the processes in which they participate. In [2] the creation of a pair of W-bosons in a constant magnetic field was considered. In [3, 4] investigations were carried out on the electromagnetic properties of a massive Dirac neutrino in a constant external field that are due to the decay of the neutrino into a virtual electron and a virtual W-boson. We notice that in the investigation of these electroweak processes methods were used that were developed in quantum electrodynamics in external fields [5, 6].

In this work we calculate the electronic contribution to the elastic scattering amplitude of a Z boson in a constant homogeneous magnetic field. Its imaginary part determines the probability for the creation of an e^+e^- pair. The essential feature of this process is that it also takes place in the absence of a field. The main laws of processes like this were investigated in [7].

The amplitude P of the elastic scattering of a Z boson with momentum $\kappa^\mu = (\kappa_0, \boldsymbol{\kappa})$ and a 4-vector polarization e^μ ($e\kappa = 0$) is expressed in terms of the polarization operator of the Z-boson $P_{\mu\nu}$ in the following way*:

*We use the system of units $\hbar = c = 1$, $e^2/4\pi = 1/137$, the metric $(+ - - -)$, γ^μ are the Dirac matrices, and $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$.

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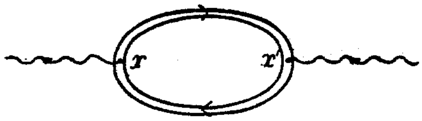

Fig. 1

$$P = (2\kappa_0 VT)^{-1} e^{*\mu} e^{\nu} P_{\mu\nu}(\kappa), \tag{1}$$

where VT is the 4-volume of the interaction.

We are interested in the one-loop electronic contribution, which corresponds to the diagram shown in Fig. 1. The continuous double lines correspond to the electron propagator S(x, x') in an external magnetic field of strength H, whose 4-potential we take to be $A_\mu = (0, 0, -Hx, 0)$. The electron propagator S satisfies the equation

$$(\gamma^\mu (i\partial_\mu - eA_\mu) - m) S(x, x') = \delta^{(4)}(x - x'). \tag{2}$$

We choose the interaction Lagrangian of the Z-boson with the electrons to be of the standard form [8]

$$L_{\text{int}} = \bar{e}\gamma^\mu (g_V + g_A \gamma^5) e Z_\mu, \tag{3}$$

where $g_V = g(1 - 2\cos 2\theta_W)/(4\cos\theta_W)$, $g_A = -g/(4\cos\theta_W)$, $G = g^2\sqrt{2}/8m_W^2 = 10^{-5}/m_p^2$ is the Fermi constant, and θ_W is the Weinberg angle. Then the contribution of the diagram is

$$P = -i\frac{e_\mu^* e_\nu}{2\kappa_0 VT} \int d^4x d^4x' \exp(i\kappa(x - x')) \times$$
$$\times \operatorname{Sp}(\gamma^\mu (g_V + g_A\gamma^5) S(x, x') \gamma^\nu (g_V + g_A\gamma^5) S(x', x)). \tag{4}$$

Using the known explicit form of the propagator solution of Eq. (2) (see, e.g., [7]) we integrate (4) over the coordinates (x, x') and average the Z-boson polarizations [this is achieved by replacing $e_\mu^* e_\nu$ by $(g_{\mu\nu} - \kappa_\mu\kappa_\nu/m_Z^2)/3$]. As a result we obtain

$$P = (8\pi\kappa_0)^{-1} \int_0^1 du \int_0^\infty \frac{d\rho}{\sin\rho} \exp(-i\Lambda\rho + i\varphi) \times$$
$$\times \{(g_V^2 + g_A^2) F_1 + (g_V^2 - g_A^2) F_2\}. \tag{5}$$

Here

$$\begin{aligned} F_1 = {} & \kappa_\parallel^2 u(1-u)\cos\rho(1-2u) + (2u(1-u) \times \cos\rho - \\ & - (u \sin 2\rho(1-u) + (1-u)\sin 2\rho u)/\sin\rho + \\ & + 2\sin\rho u \sin\rho(1-u)/\sin^2\rho)\, \kappa_\parallel^2 \kappa_\perp^2/(3m_z^2) + (-2 + \kappa_\parallel^2/m_z^2) \times \\ & \times \left(\frac{2}{3} u(1-u)\kappa_\parallel^2 \sin\rho u \sin\rho(1-u) - \frac{ih}{3\rho}\cos\rho(1-2u) + \right. \\ & \left. + \frac{ih}{3\sin\rho}\right) - \frac{ih}{\rho}\cos\rho(1-2u) - \kappa_\perp^2 \sin\rho u \sin\rho(1-u)/\sin^2\rho, \\ F_2 = {} & m^2\left(\cos\rho + \frac{2}{3}\sin\rho u \sin\rho(1-u)\cdot(2 + \kappa_\perp^2/m_z^2)\right), \\ \varphi = {} & \nu(\rho u(1-u) - \sin\rho u \sin\rho(1-u)/\sin\rho), \\ \Lambda = {} & (m^2 - m_z^2 u(1-u))/h. \end{aligned} \tag{6}$$

We have introduced the following parameters:

$$h = eH,\ \lambda = m_z^2/m^2,\ \nu = \kappa_\perp^2/h;$$

$\kappa_\parallel^2 = \kappa_0^2 - \kappa_3^2$; κ_3 is the longitudinal component of the Z-boson momentum; $\kappa_\perp^2 = \kappa_1^2 + \kappa_2^2$; $\kappa_{1,2}$ are the transverse components of the momentum. The expression (5) is the elastic scattering amplitude of a Z-boson in an external magnetic field. Its imaginary part is related to the decay probability of $Z \to e^+e^-$ in the following way: $w = 2\,\mathrm{Im}\,P$.

The exact integration of (5) does not seem to be possible. Therefore, we consider various limiting cases.

We consider the decay probability at large transverse Z-boson momenta $\kappa_\perp$ in fairly weak fields $H \ll H_0 = m^2/e$. At $H \ll H_0$ and $\kappa_\perp \gg m_Z$ the motion is quasiclassical and the main contribution to the integral (5) comes from the region $\rho \ll 1$. Expanding the trigonometric functions with respect to ρ in the integrand of (5) and keeping only the leading terms we can carry out the integration with respect to ρ. As a result we obtain the following expression:

$$w = m^2 (2\pi^{3/2}\kappa_0)^{-1} \int\limits_0^{1/4} \frac{dx}{\sqrt{1-4x}} \Big\{ [g_V^2 + g_A^2] \cdot \Big[(2\lambda x - 1)\,\Phi_1(z) - \frac{2}{3}\frac{\varkappa^{2/3}}{x^{1/3}}\Phi'(z) + \frac{2}{9}\frac{\varkappa^{4/3}}{\lambda x^{2/3}}(1-3x)\,\Phi(z) \Big] + (g_V^2 - g_A^2)\Big[\Phi_1(z) + \frac{2}{3}\frac{\varkappa^{2/3}}{\lambda x^{1/3}}\Phi'(z)\Big]\Big\}, \tag{7}$$

where $\Phi(z) = \pi^{-1/2}\int\limits_0^\infty dt \cos(zt + t^3/3)$ and $\Phi'(z)$ are the Airy function and its derivative, $\Phi_1(z) = \int\limits_z^\infty dx\,\Phi(x)$, the argument $z = (1 - \lambda x)/(\ x)^{2/3}$, $x = u(1 - u)$. Here $\varkappa$ is the parameter characteristic for processes in an external field [5-7], which determines the dependence of the probability (7) on the external field (the only paramater at $H \ll H_0$):

$$\varkappa = \frac{\kappa_\perp}{m}\frac{H}{H_0} = \frac{e}{m^3}\left[-(F^{\mu\nu}\kappa_\nu)^2\right]^{1/2}. \tag{8}$$

We consider the asymptotic behavior of the probability at small and large $\varkappa$. At $\varkappa \ll 1$ the main contribution to (7) comes from the vicinity of the points $z = 0$ and $z = z_0$, where

$$z_0 = (\lambda/4 - 1)/(\varkappa/4)^{2/3}. \tag{9}$$

Using the method given in [7] we obtain an asymptotic series in powers of the parameter $\varkappa$. However, we restrict ourselves to the first corrections in $\varkappa$ that enable us to obtain the characteristic features of this process. As a result we obtain

$$w = \frac{m^2}{4\pi\kappa_0}\Big\{\Big[\frac{1}{3}(1-4\lambda)^{1/2}(\lambda - 1) - \frac{8}{9}\frac{\varkappa^2}{\lambda^2} + \frac{\varkappa}{2\sqrt{3\lambda}}\cos\Big(\frac{\lambda^{3/2}}{3\varkappa}\Big)\Big](g_V^2 + g_A^2) + (g_V^2 - g_A^2) \times \Big[(1-4/\lambda)^{1/2} + \frac{8}{3}\frac{\varkappa^2}{\lambda^3} - \frac{5}{\sqrt{3}}\frac{\varkappa}{\lambda^{3/2}}\cos\Big(\frac{\lambda^{3/2}}{3\varkappa}\Big)\Big]\Big\}. \tag{10}$$

The first term in (10) is independent of the field and is exactly the probability of a free decay. We pay special attention to the appearance (apart from the usual term in $\varkappa^2$) of the terms of the form $\varkappa\cos(\lambda^{3/2}/3\varkappa)$. They cause oscillations of the decay as $\varkappa$ changes. The appearance of oscillations in decays in an external field has been demonstrated many times (see, e.g., [7, 10, 11]). We note that the oscillations only arise in those reactions that take place in the absence of a field. In this case the argument of the Airy function in the expression for the probability of the process extends along the negative direction not to $-\infty$ but to the given point z_0, whose vicinity gives the sought-after oscillations.

For the probability (7) at $\varkappa \gg 1$ we have $z \ll 1$ in the important region, and by setting $\Phi(z) \simeq \Phi(0)$, we find the asymptotics

$$w = \frac{m^2}{(2\pi)^3}\frac{7}{270}\frac{(3\varkappa)^{4/3}}{\kappa_0\lambda}\Gamma^4(1/3)[g_V^2 + g_A^2]. \tag{11}$$

We note that the oscillations of probability for varying $\varkappa$ also arise in the case of small transverse momenta. This is related to the discreteness of the spectrum of the transverse motion of an electron in the magnetic field. The period of the oscillations, as in the case of large transverse Z-boson momenta, depends on the external field as 1/H.

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