Milky Way's young populations: distance scale and kinematics of maser sources and Cepheids

Rotation curve of the Galaxy and its spiral pattern from new data on Milky Way masers and Cepheids

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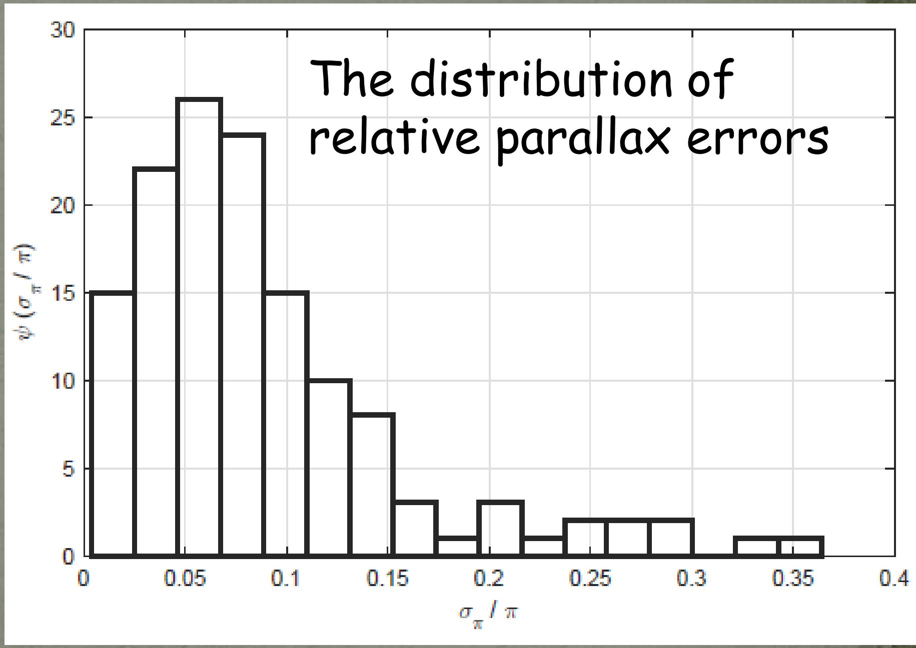
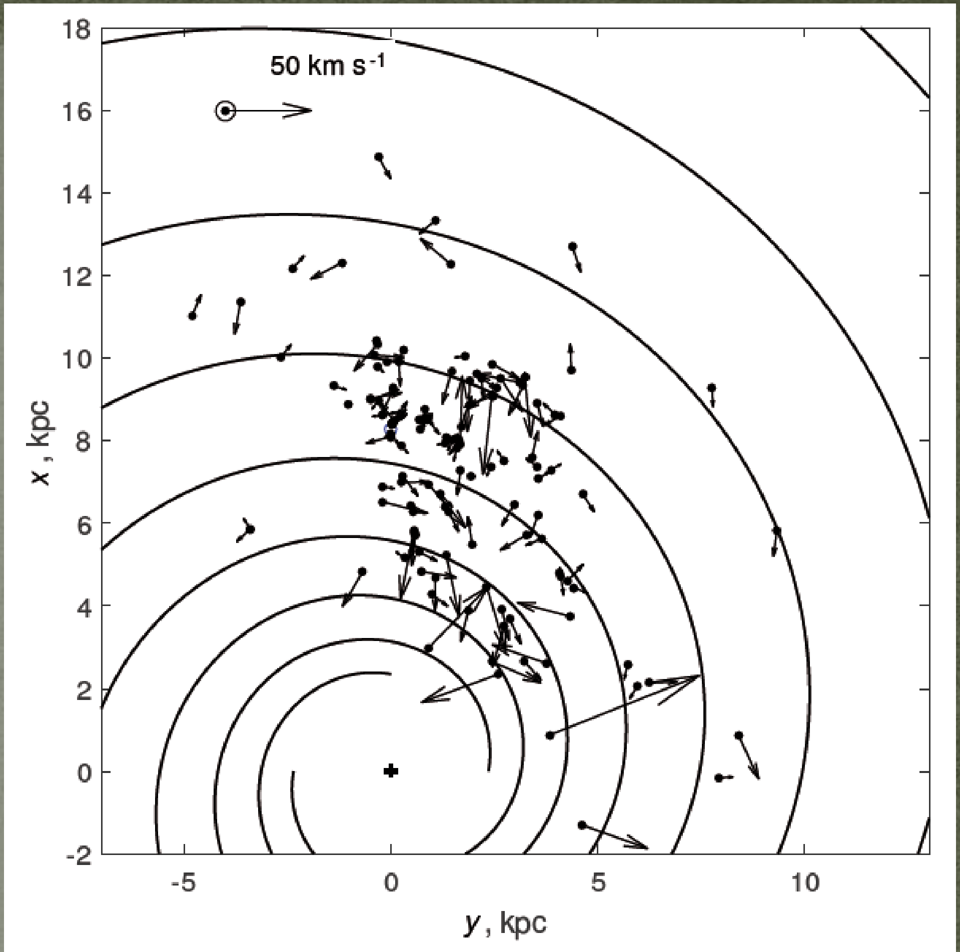
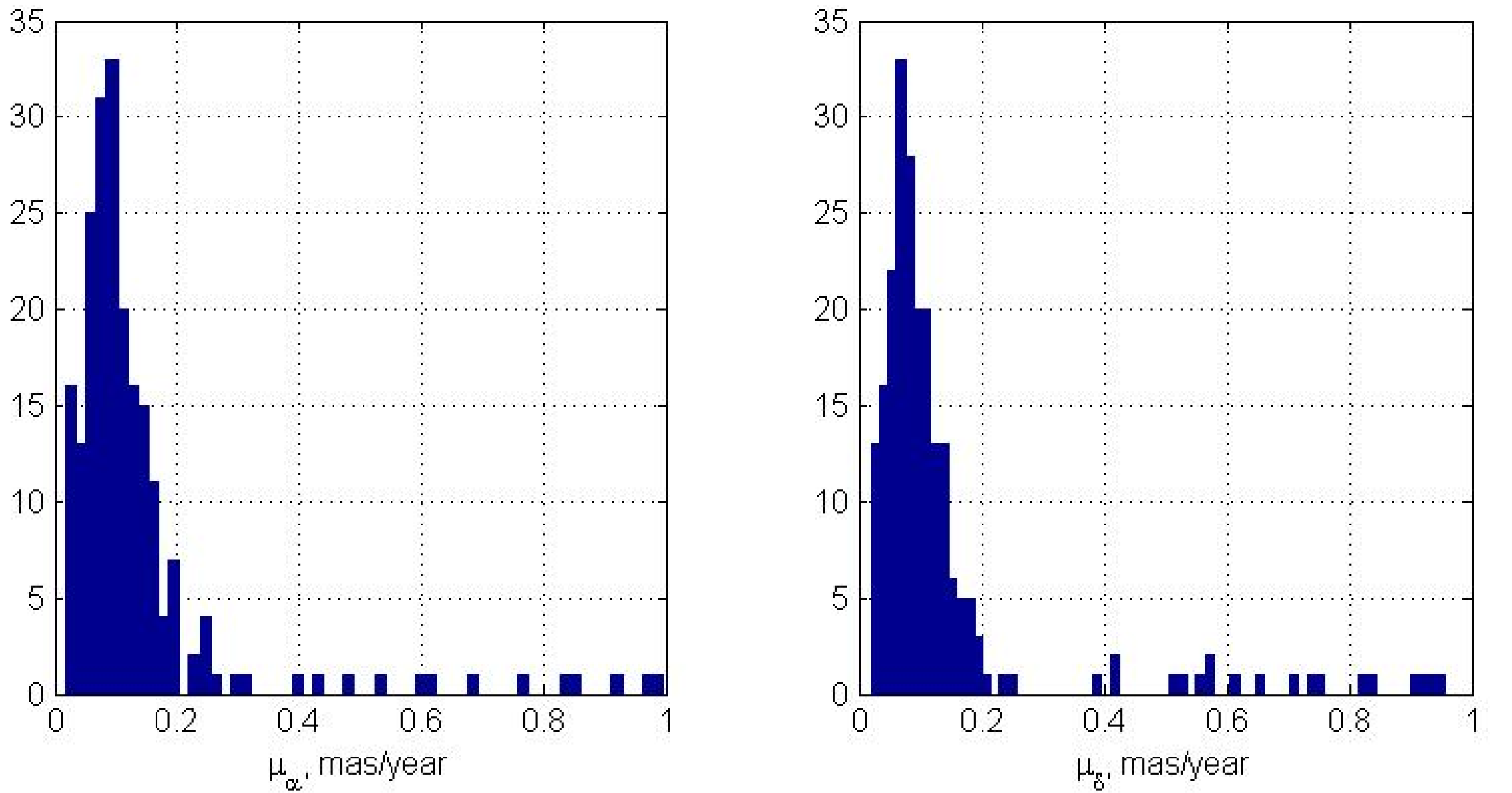
- **Maser sources in massive star-formation regions**
  Most extended sample of maser sources: Reid et al. (2014) list (~100 sources) + additional 40 sources taken from recent papers (water – methanol)
- Trigonometric distances, proper motions and radial velocities – VERA / VLBA observations
- Typical accuracy of radial velocities ~3-5 km/s
- Wide range of the galactocentric distances: from ~1 to 15 kpc



- Most maser sources populate I / II galactic quadrants – may result in the selection effects
- Kinematic four spiral arms are shown, with residual velocity vectors relative to pure rotation model



- **TGAS**: Tycho-GAIA Astrometric solution (GAIA Collaboration, A&A, V.595, A1-A7, A133, 2016)
- **New proper motions:** ~225 Cepheids
- **Initial distances:** PL relation based on 9 Cepheids, members of open clusters (Berdnikov et al. 1996)
- **Radial velocities:** for ~150 Cepheids – Radial Velocity Meter database (Gorynya, Rastorguev, Samus at al., 1987-2016); published data (incuding Fernie database 1995+)
- **Method:** statistical parallax maximum-likelyhood technique (Zabolotskikh et al. 2002, Rastorguev et al. 2017), most detailed treating of random errors in data and *systematic errors of the distance scale*



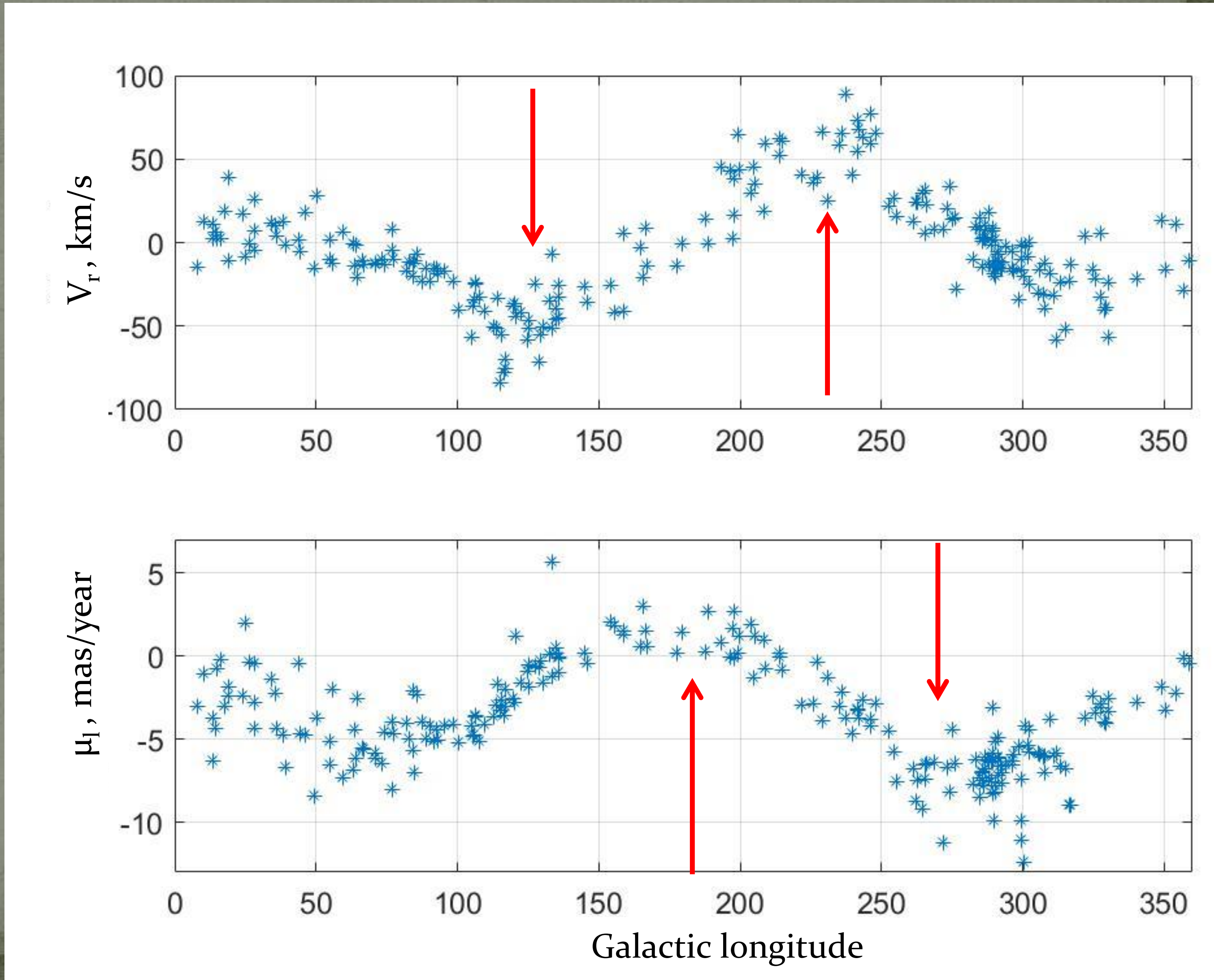
TGAS distribution of proper motion errors

At the characteristic distance of 2 kpc mean accuracy of the tangential velocity – approx. 1 km/s - is comparable to the accuracy of radial velocities

Some errors are as large as 3-4 mas/year

Classical "double-wave" in radial velocities and proper motions (TGAS) of Cepheids (age < 300 Myr)

Phase shift
Δφ ≈ ¼ π
between
"waves"



The models of space velocity field

$$\begin{pmatrix} V_r \\ kr\mu_l \\ kr\mu_b \end{pmatrix} - \begin{pmatrix} R_0(\omega - \omega_0)\sin l \cos b \\ (R_0 \cos l - r\cos b)(\omega - \omega_0) - r\omega_0 \cos b \\ -R_0(\omega - \omega_0)\sin l \sin b \end{pmatrix}$$

$$-G^T \times \begin{pmatrix} U_0 \\ V_0 \\ W_0 \end{pmatrix} = \delta\vec{V}_{\rm loc}, \qquad (4)$$

- **Model #1: pure rotation**
- (all formulae from Rastorguev et al., AstBull, V.72, pp. 122-140, 2017)

The models of space velocity field

$$\begin{pmatrix} V_r \\ kr\ \mu_l \\ kr\ \mu_b \end{pmatrix} - \begin{pmatrix} R_0(\Omega-\Omega_0)\sin l\cos b \\ (R_0\cos l - r\cos b)(\Omega-\Omega_0) - r\Omega_0\cos b \\ -R_0(\Omega-\Omega_0)\sin l\sin b \end{pmatrix}$$

$$- \begin{pmatrix} -(R_0(\frac{\Pi}{R} - \frac{\Pi_0}{R_0})\cos l - \frac{\Pi}{R}r\cos b)\cos b \\ R_0(\frac{\Pi}{R} - \frac{\Pi_0}{R_0})\sin l \\ (R_0(\frac{\Pi}{R} - \frac{\Pi_0}{R_0})\cos l - \frac{\Pi}{R}r\cos b)\sin b \end{pmatrix}$$

$$-G^T \times \begin{pmatrix} U_0 \\ V_0 \\ W_0 \end{pmatrix} = \delta\vec{V}_{loc}, \qquad (7)$$

Model #2: rotation + spiral density wave perturbations

Rotation matrix G

$$G = \begin{pmatrix} \cos b \cos l & -\sin l & -\sin b \cos l \\ \cos b \sin l & \cos l & -\sin b \sin l \\ \sin b & 0 & \cos b \end{pmatrix} \qquad (5)$$

transforms the velocity components

$$\vec{V}_{\rm loc} = \begin{pmatrix} V_r \\ kr\,\mu_l \\ kr\,\mu_b \end{pmatrix} \qquad (6)$$

from "local" coordinate system (connected with the direction to the object) to Descart system

$$\vec{V}_{\rm gal} = \begin{pmatrix} U \\ V \\ W \end{pmatrix} = G \times \vec{V}_{\rm loc}$$

- Modified angular velocity

$$\Omega = \omega + \frac{\Theta}{R}, \;\; \Omega_0 = \omega_0 + \frac{\Theta_0}{R_0},$$

- Perturbations due to spiral pattern in linear approximation (Lin and Shu 1969) for the object and the Sun:

$$\begin{pmatrix} \Pi \\ \Theta \end{pmatrix} = \begin{pmatrix} f_R \cdot \cos\chi \\ f_\Theta \cdot \sin\chi \end{pmatrix}, \;\; \begin{pmatrix} \Pi_0 \\ \Theta_0 \end{pmatrix} = \begin{pmatrix} f_R \cdot \cos\chi_0 \\ f_\Theta \cdot \sin\chi_0 \end{pmatrix}$$

- Phase angle in the wave

$$\chi - \chi_0 = m(\psi - \mathrm{ctg}\, i \times \lg \frac{R}{R_0})$$

- Substituting angular velocities in Bottlinger equations

$$\begin{aligned} \omega \rightarrow \Omega &= \omega + f_\Theta \times \sin\chi / R, \\ \omega_0 \rightarrow \Omega_0 &= \omega_0 + f_\Theta \times \sin\chi_0 / R_0 \end{aligned}$$

Covariation Matrix

- Residual velocity:

$$\vec{\delta} V_{\rm loc} = \begin{pmatrix} \delta V_r \\ kr\ \delta\mu_l \\ kr\ \delta\mu_b \end{pmatrix}$$

- Error and correlations matrix

$$L_{\rm err} = \langle \delta\vec{V}_{\rm loc} \cdot \delta\vec{V}^T_{\rm loc} \rangle$$
$$= \begin{pmatrix} \sigma^2_{V_r} & 0 & 0 \\ 0 & k^2r^2\ \sigma^2_{\mu_l} & k^2r^2\ \sigma_{\mu_l}\ \sigma_{\mu_b}\ \rho_{\mu_l\ \mu_b} \\ 0 & k^2r^2\ \sigma_{\mu_l}\ \sigma_{\mu_b}\ \rho_{\mu_l\ \mu_b} & k^2r^2\ \sigma^2_{\mu_b} \end{pmatrix}$$

- "Cosmic" velocity dispersion (peculiar velocity ellipsoid in proper axes)

$$L_0 = \begin{pmatrix} \sigma U^2 & 0 & 0 \\ 0 & \sigma V^2 & 0 \\ 0 & 0 & \sigma W^2 \end{pmatrix}$$

- Transformation of the velocity ellipsoid to "local" system

$$L_{\rm resid} = P \times G_S \times L_0 \times G_S^T \times P^T$$

- Auxiliary vector

$$\vec{\Upsilon} = M \times [G^T \times \vec{V}_0 + \vec{V}_{\rm sys}] - r/p \times P \times \partial\vec{V}_{\rm sys}/\partial r$$

- $p = r_{expected} / r_{true}$ – distance scale factor

- The covariance matrix of the distance errors

$$\delta L = (\sigma_\pi/\pi)^2 \times [M \times G_S \times L_0 \times G_S^T \times M^T + \vec{\Upsilon} \times \vec{\Upsilon}^T]$$

- *(M, P)* – auxiliary matrices 3x3

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}$$

- Full covariation matrix for residual velocity: extended "weight" of the contribution of each sample object to the likelihood function

$$L_{\rm loc} = L_{\rm err} + L_{\rm resid} + \delta L.$$

- Full velocity of systematic motions

$$\vec{V}_{\text{sys}} = \vec{V}_{\text{rot}} + \vec{V}_{\text{spir}}$$

- The calculation of rotation space velocity field errors induced by random and systematic errors of the distances

$$\frac{\partial \vec{V}_{\text{rot}}}{\partial r} = \begin{pmatrix} R_0 \times \frac{\partial}{\partial r}(\omega - \omega_0) \times \sin l \times \cos b \\ (R_0 \times \cos l - r \times \cos b) \times \frac{\partial}{\partial r}(\omega - \omega_0) - \omega \times \cos b \\ -R_0 \times \frac{\partial}{\partial r}(\omega - \omega_0) \times \sin l \times \sin b \end{pmatrix} \quad (22)$$

- where

$$\frac{\partial(\omega - \omega_0)}{\partial r} \approx \frac{\cos b}{R}(r \cos b - R_0 \cos l) \times \sum_{n=1}^{K} \frac{1}{(n-1)!} \frac{\partial^n \omega_0}{\partial r^n}(R - R_0)^{n-1},$$

$$\frac{\partial \vec{V}_{\rm spir}}{\partial r} = \left(\frac{\partial V_r^{\rm sp}}{\partial r} \; \frac{\partial V_l^{\rm sp}}{\partial r} \; \frac{\partial V_b^{\rm sp}}{\partial r} \right)^T$$

- The same for the contribution of spiral density wave perturbations

$$\frac{\partial V_r^{sp}}{\partial r} = f_R/R \cdot \cos b \cdot (\cos b \cos \chi +$$

$$+ D \cdot (\cos \chi \cdot \frac{\partial R}{\partial r}/R + \sin \chi \cdot \frac{\partial \chi}{\partial r}))-$$

$$- f_\Theta R_0/R \cdot \sin l \cos b \cdot (\cos \chi \cdot \frac{\partial \chi}{\partial r} - \sin \chi \cdot \frac{\partial R}{\partial r}/R);$$

$$\frac{\partial V_l^{sp}}{\partial r} = - f_R R_0/R \cdot (\sin \chi \cdot \frac{\partial \chi}{\partial r} + \cos \chi \cdot \frac{\partial R}{\partial r}/R) \cdot \sin l +$$

$$+ f_\Theta/R \cdot (\cos b \sin \chi + D \cdot (\sin \chi \cdot \frac{\partial R}{\partial r}/R - \cos \chi \cdot \frac{\partial \chi}{\partial r}));$$

$$\frac{\partial V_b^{sp}}{\partial r} = - \frac{\partial V_r^{sp}}{\partial r} \cdot \tan b.$$

- Probability density of the residual velocity distribution for single star

$$f(\delta\vec{V}_{\rm loc} \mid \Lambda)$$
$$= (2\pi)^{-3/2}\,|L_{\rm loc}|^{-1/2}\exp\{-\frac{1}{2}\delta\vec{V}^{T}_{\rm loc}L^{-1}_{\rm loc}\delta\vec{V}_{\rm loc}\}$$

- Full probability density for the whole sample

$$F(\delta\vec{V}_{\rm loc}(1),\cdots,\delta\vec{V}_{\rm loc}(N) \mid \Lambda) = \prod_{i=1}^{N} f_i(\delta\vec{V}_{\rm loc} \mid \Lambda)$$

- The likelihood function depends on Λ - the vector of unknown parameters describing the velocity field

$$LF(\Lambda) = \frac{3}{2}N\ln 2\pi$$
$$+\frac{1}{2}\sum_{i=1}^{N}[\ln|L_{\rm loc}(i)| + \delta\vec{V}^{T}_{\rm loc}(i)L_{\rm loc}(i)^{-1}\delta\vec{V}_{\rm loc}(i)]$$

Some additional constraints

- 2 models of the velocity field (pure rotation / rotation + perturbation from spiral density wave)
- Radial velocity dispersion depends on the galactocentric distance: three variants (see next slide)
- The ratio of the two horizontal velocity ellipsoid axes depends on the galactocentric distance (Lindblad relation !) and controls by current values of the angular frequency ω and epicyclic frequency κ

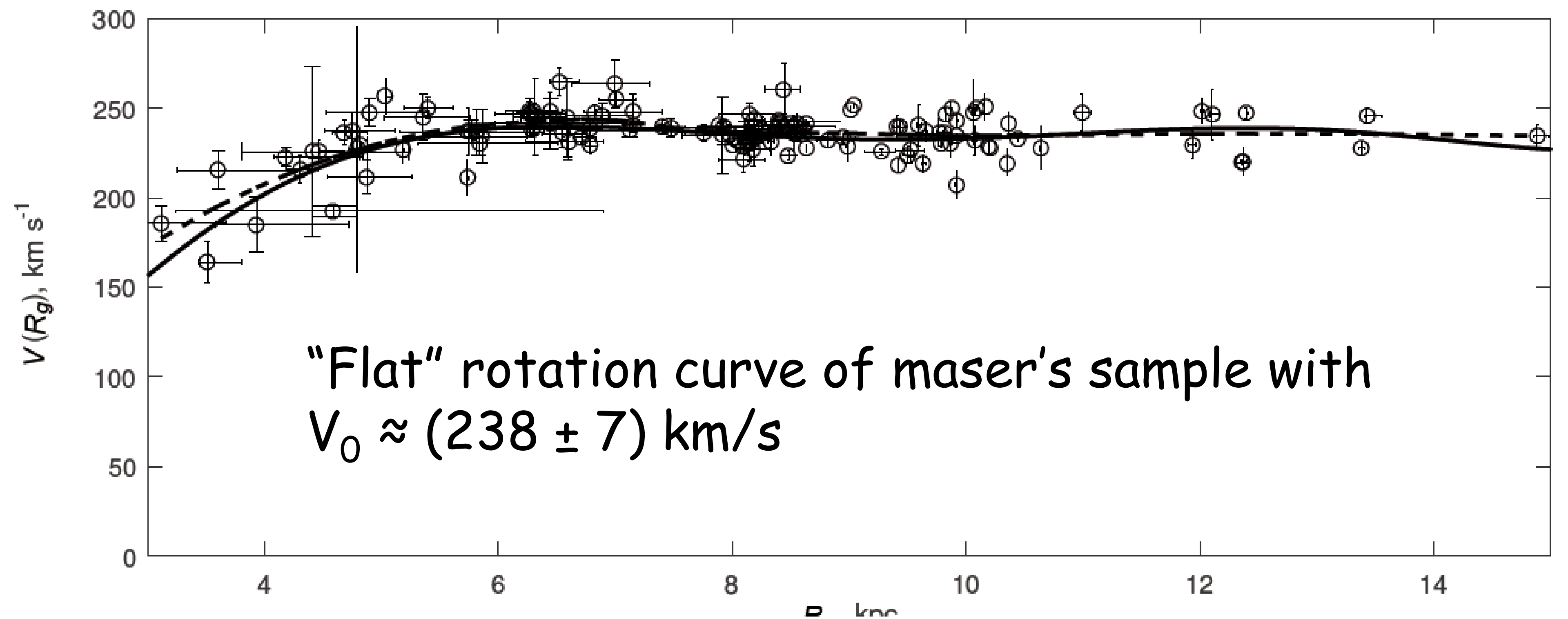
- **The variation of radial velocity dispersion with the galactocentric distance:**
  - 1) constant dispersion $$\sigma U(R) = \sigma U(R_0) = \sigma U0$$
  - 2) exponential decrease $$\frac{\sigma U(R)}{\sigma U(R_0)} \approx \exp\left(\frac{R_0 - R}{H_D}\right)$$
  - 3) Toomre-like "equation of state" $$\frac{\sigma U(R)}{\sigma U(R_0)} \approx \frac{\kappa(R_0)}{\kappa(R)} \exp\left(\frac{R_0 - R}{H_D}\right)$$
- **Disk exponential scale adopted H_D = 3, 4 кпк (small effect on the results)**
- **Calculate correction to the distance scale factor (with typical accuracy $\sigma_p \sim \pm 0.02$-0.03)**

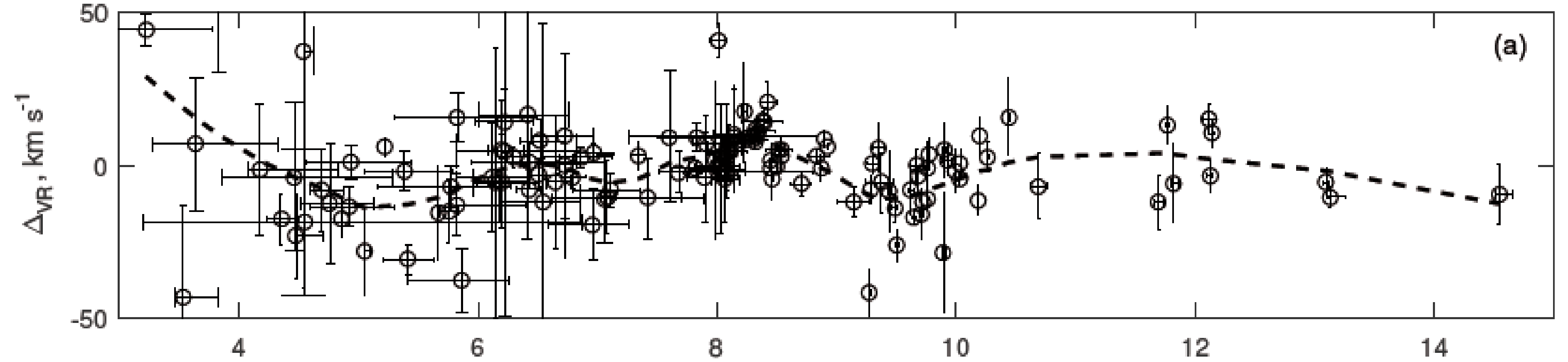
- Maser kinematical parameters

- Best model: four spiral arms, constant radial and vertical velocity dispersions

- Significant radial and tangential amplitudes of spiral perturbations

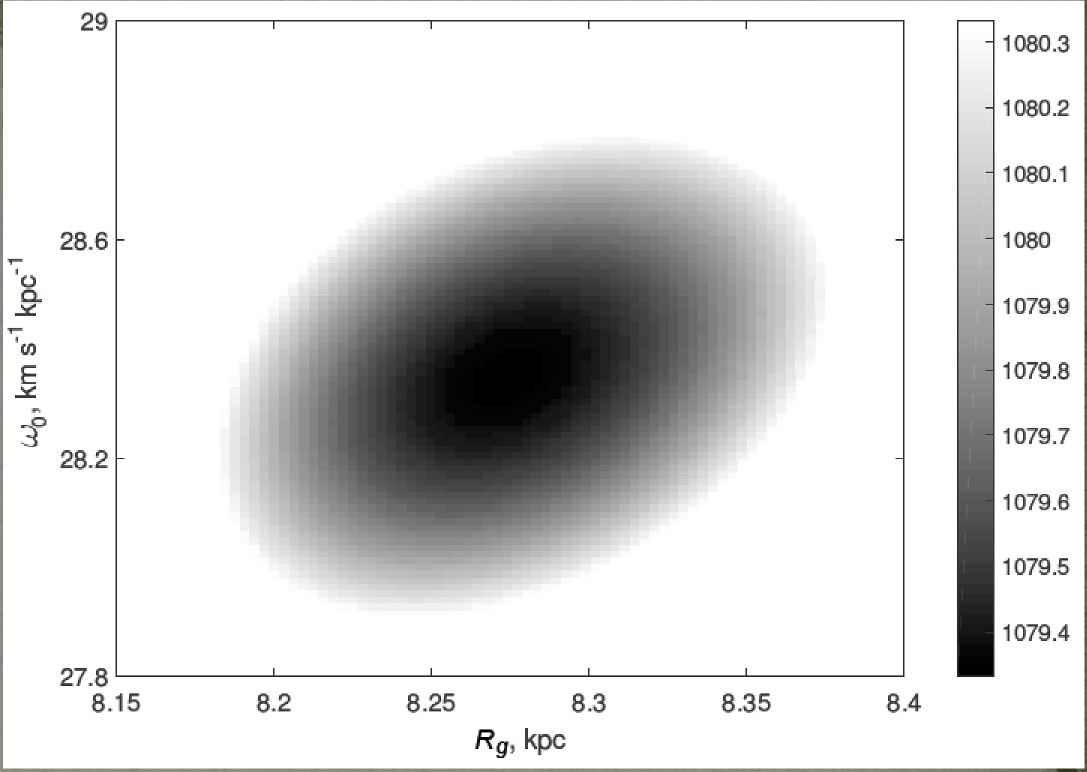
p	0.961 ± 0.020
R_0, kpc	8.27 ± 0.13
U_0, km s^{-1}	-10.98 ± 1.40
V_0, km s^{-1}	-19.62 ± 1.15
W_0, km s^{-1}	-8.93 ± 1.05
$\sigma U0$, km s^{-1}	9.43 ± 0.88
$\sigma W0$, km s^{-1}	5.86 ± 0.80
f_R, km s^{-1}	-7.00 ± 1.48
f_Θ, km s^{-1}	2.62 ± 1.05
χ_0, deg	130.3 ± 10.8
i, deg	-10.39 ± 0.25
ω_0, km s^{-1} kpc^{-1}	28.35 ± 0.45
$d\omega/dR$, km s^{-1} kpc^{-2}	-3.83 ± 0.08
$d^2\omega/dR^2$, km s^{-1} kpc^{-3}	1.17 ± 0.05
$d^3\omega/dR^3$, km s^{-1} kpc^{-4}	-0.08 ± 0.04
$d^4\omega/dR^4$, km s^{-1} kpc^{-5}	-0.30 ± 0.03
$LF_{\min}$	1079.2939





Residuals of the radial velocity component from pure rotation model: large radial amplitude of the density wave perturbations

- Calculation of the errors:
  cross-section of the LF profile near global min LF_0 by the "surface" $LF = Lf_0 + 1$ projected onto $R_0 - \omega_0$ plane: correlation of the parameters



- Other correlations: *($R_0 - \omega_0'$), ($R_0 - p$), etc.*

Additional constrains for Cepheids

- Separate calculations for Cepheids with $P > 10^d$ (68 stars) and $P < 10^d$ (157 stars)
- R_0 = 8.2 kpc adopted

Distance scale factor $p = r_{expected} / r_{true}$ (3 cases) for long- and short period Cepheid samples

Const σU	Rotation	Ratio	Rot + Spir	Ratio
Long	0.924	1.12	0.943	1.13
Short	0.823		0.837	
Exp σU	Rotation	Ratio	Rot + Spir	Ratio
Long	1.067	1.04	1.051	1.08
Short	0.978		0.977	
Toomre σU	Rotation	Ratio	Rot + Spir	Ratio
Long	1.020	1.16	1.012	1.13
Short	0.883		0.894	

Distance scales

- In most cases $p_{short} < p_{long}$:
- Distance scale of short-period Cepheids is systematically (by 5…15%) shorter as compared to long-period group
- **Possible reason:** unidentified overtone pulsators in short-period group: distance underestimate may reach ~60% because of "wrong" PL + extinction used (Zabolotskikh et al. 2002)

We made a "Grand Unification":

- **Basic idea:** to adjust the distance scale of the two samples in accord with proper values of the scale factors found separately, as

$$(r_{new} = r_{expected} / p)$$

- The resulting large sample has (presumably) consistent distances and can by analyzed by the same technique (maximum-likelihood solution)

New distance scale factors, p_{new}

Model	Rotation	Rot + Spir
Constant σU	0.990	0.987
Exponential σU	0.965	0.999
Toomre-like σU	0.984	0.987

Within the errors (σ_p ~ ±0.02) the scales are nearly balanced and very close to unity

Pure rotation parameters (adopting distance scale factor *p = 1*) (likelihood function values at the minimum, LF0, are also given)

Model/LF0	U0 km/s	V0 km/s	W0 km/s	σU km/s	σW km/s	Ω0 km/s/kpc	ω' km/s/kpc²
Const σU (2549) (The best model)	-9.1	-12.4	-7.5	13.8	7.6	28.9	-3.94
Exp σU (2577)	-7.2	-14.1	-6.5	17.2	6.4	30.5	-5.0
Toomre σU (2569)	-8.7	-11.0	-7.0	15.3	7.1	29.7	-3.84
Typical errors	±2.0	±1.5	±1.0	±1-2	±0.70	±0.8	±0.3

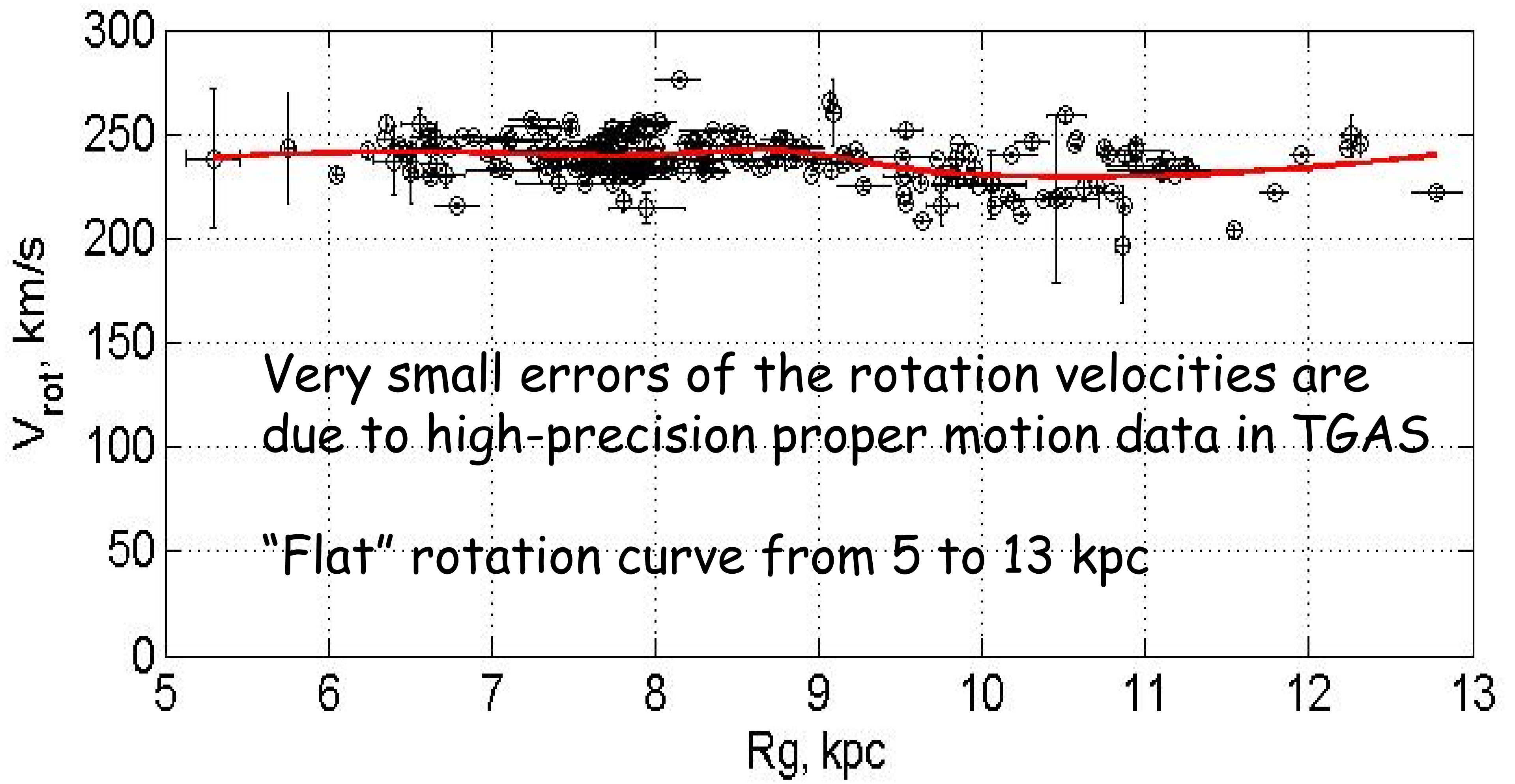
The parameters of composite model – rotation and the density wave (adopting distance scale factor *p=1)* (with minimal values of the likelihood function LF0)

Model /LF0	U0 km/s	V0 km/s	W0 km/s	σU km/s	σW km/s	ω0 km/s/ kpc	ω' km/s/ kpc^2	fR km/s	fθ km/s	X0 deg	i deg
Const σU 1932 (The best model)	-13.1	-13.8	-7.4	13.8	7.5	29.1	-4.2	-4.0	+1.8	159	-9.6
Exp σU 1955	-12.5	-14.4	-6.5	16.8	6.5	31.1	**-5.1**	-5.0	+2.9	163	-10.3
Toomre σU 1940	-12.0	-12.4	-6.9	15.9	7.0	30.3	-4.2	-4.0	+1.7	147	-10.4
±	±1.5	±1.0	±0.8	±1.2	±0.6	±0.5	±0.2	±2.0	±1.5	±25	±1.2

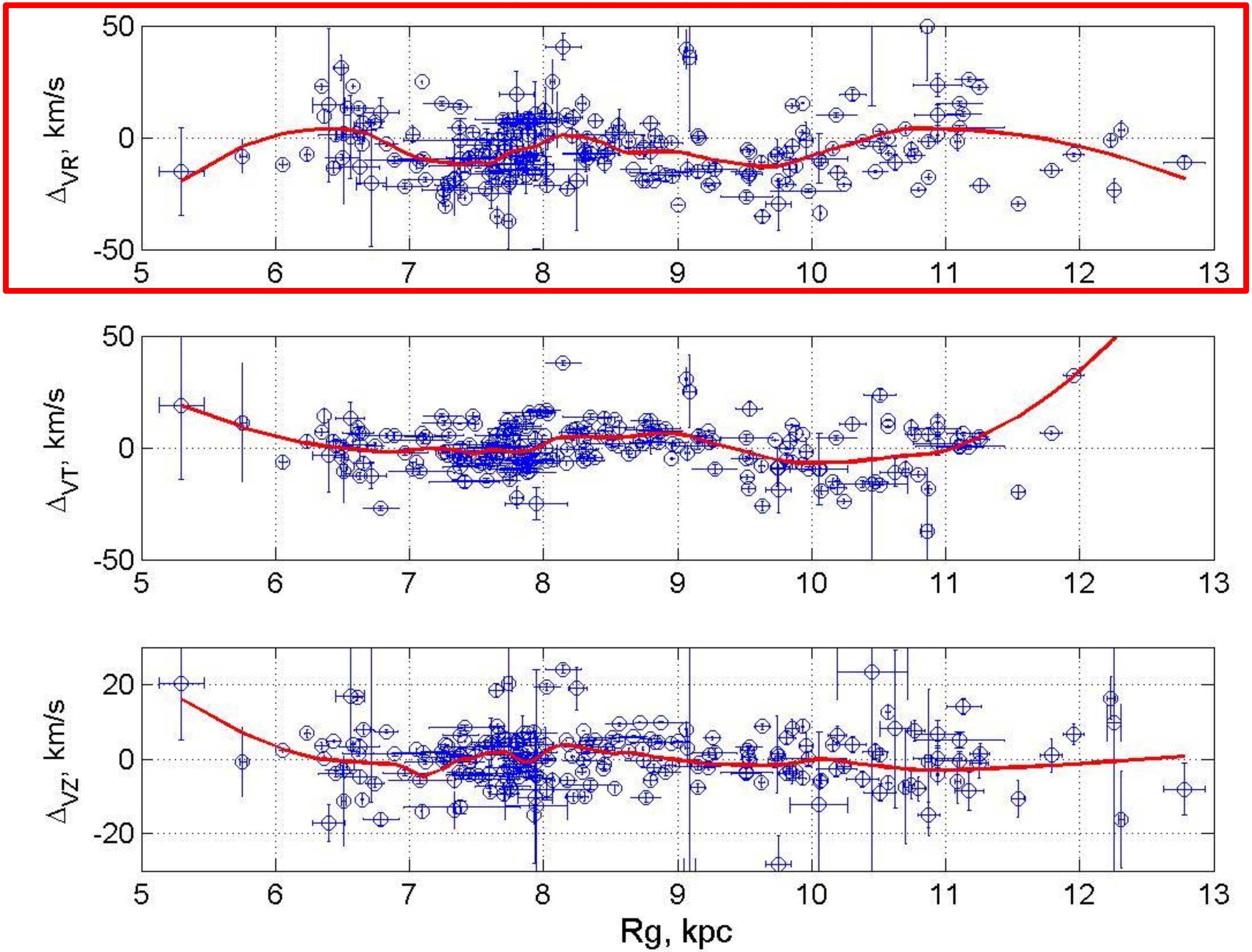
- From minimal values of the likelihood function (LF0) we conclude that the best model of pure rotation correspond to the constant radial velocity dispersion
- All velocity field models with the spiral density wave perturbations fit the observations much better than pure rotation models
- The best model includes rotation and spiral perturbations as well as constant radial dispersion (LF0 ≈ 1932 as compared to the model with pure rotation with LF0 ≈ 2549) (the same was shown for maser sample by Rastorguev et al. 2017)

Rotation curve for Cepheid combined sample

$V_0 \approx (238 \pm 8)$ km/s, $A_0 \approx (17.2 \pm 1.2)$ km/s/kpc



Residuals from pure rotation



Comparing the kinematical parameters of Cepheids and galactic masers (Rastorguev et al. 2017)

	U0	V0	W0	σU0	σW0	ω0	ω'	fR	fθ	x0	i
Ceps	-13.1	-13.8	-7.4	13.8	7.5	29.1	-4.2	-4.0	+1.8	159	-9.6
±	±1.5	±1.0	±0.8	±1.2	±0.6	±0.5	±0.2	±2.0	±1.5	±25	±1.0
Ma-sers	-11.0	-19.6	-8.9	9.4	5.9	28.4	-3.8	-7.0	+2.6	130	-10.4
±	±1.4	±1.2	±1.1	±0.9	±0.8	±0.5	±0.1	±1.5	±1.1	±11	±0.3

Differences of σU0, σW0, fR, fθ, i can be explained by the differences of ages of Cepheids (from 25-30 to ~300 Myr) and maser sources (less than 20-25 Myr)

Pitch angle is in good agreement with Dambis et al. (2016) data from space distribution of Cepheids

The problem with large differences of V0 (LSR) remains: the selection effects due to the absence of observations of masers in III-IV quadrants ? Waiting for ALMA ?

- Thank for your attention !

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