Formulation of the initial boundery velue problems in the theory of multilayer thermoelastic thin bodies in moments (part II)

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Abstract: Various representations of the equations of motion, the heat influx, the constitutive relations of physical and heat content are given for the new body domain parametrization. The definition of the kth order moment of a certain quantity with respect to an orthonormal polynomial systems is given. The expressions of moments of first- and second-order partial derivatives of a certain tensor field are obtained and this is also done for some important expressions required for constructing different variants of the thin body theory.

1. Representation of equations of motion and heat influx and constitutive relations of physical and heat contents of micropolar theory of multilayer elastic thin bodies with one small size

In what follows, for brevity, we present certain representations of equations of motion and constitutive relations in the case of a one-layer thin body, and then we show how one can obtain the desired relations using the rule presented above and write certain relations.

1.1. Representation of equations of motion and constitutive relations of physical and heat contents of the micropolar theory of one-layertic thin bodies with one small size

The new parametrization of a one-layer thin domain [1–6] is performed by the relation, which is obtained from (I: 1) under the absence of index α under the symbols. To obtain the representations of equations of motion and constitutive relations, we need the representations of the gradient and the divergence under the parametrization considered. Let us obtain the representations of these operators. Omitting the index α, from (I: 12) and (I: 24), we find

$$\mathbf{r}_p = g_p^{\bar{m}} \mathbf{r}_{\underset{m}{-}} = g_{p\underset{m}{+}} \mathbf{r}^{\overset{+}{m}}, \quad \mathbf{r}^p = g^p_{\underset{m}{-}} \mathbf{r}^{\bar{m}} = g^p_{\underset{m}{+}} \mathbf{r}^{\overset{+}{m}}, \tag{1}$$

and also from (I:1.23), we have

$$\begin{gathered} g^P_{\underset{M}{-}} = \overset{(-)}{\vartheta}{}^{-1} A^P_{\underset{M}{-}}, \quad \overset{(-)}{\vartheta} = \det(g_I^{\bar{J}}), \quad g^3_{\underset{M}{-}} = -g_P^{\bar{3}} g^P_{\underset{M}{-}}, \quad g_P^{\bar{3}} = x^3 g^{\bar{3}}_{\underset{P}{+}}, \\ g^{\bar{3}}_{\underset{P}{+}} = h^{-1}\partial_P h, \quad h = |\mathbf{h}|, \quad A^P_{\underset{M}{-}} \equiv g^{\bar{P}}_{\underset{M}{-}} + x^3 a^P_{\underset{M}{+}}, \quad a^P_{\underset{M}{+}} \equiv (g^{\bar{I}}_{\underset{I}{+}} - 1) g^{\bar{P}}_{\underset{M}{-}} - g^{\bar{P}}_{\underset{M}{+}}. \end{gathered} \tag{2}$$

Moreover, note that the following relations hold [1–6]:

$$g^{P}_{\overset{-}{M}} = \sum_{s=0}^{\infty} \underset{(s)}{A}{}^{\bar{P}}_{\overset{+}{M}} (x^3)^s, \quad \underset{(s)}{A}{}^{\bar{P}}_{\overset{+}{M}} = (g^{\bar{P}}_{\overset{-}{N_1}} - g^{\bar{P}}_{\overset{+}{N_1}}) \cdot \ldots \cdot (g^{\bar{N}_{s-1}}_{\overset{-}{M}} - g^{\bar{N}_{s-1}}_{\overset{+}{M}}), \quad \underset{(0)}{A}{}^{\bar{P}}_{\overset{+}{M}} = g^{\bar{P}}_{\overset{-}{M}}. \tag{3}$$

By the first and third relations in (2) and the second relation in (1), we find that

$$\mathbf{r}^P = g^{P}_{\overset{-}{M}} \mathbf{r}^{\bar{M}}, \quad \mathbf{r}^3 = g^{3}_{\overset{-}{M}} \mathbf{r}^{\bar{M}} + \mathbf{r}^{\bar{3}} = \mathbf{r}^{\bar{3}} - g^{\bar{3}}_{P} \mathbf{r}^P = \mathbf{r}^{\bar{3}} - g^{\bar{3}}_{P} g^{P}_{\overset{-}{M}} \mathbf{r}^{\bar{M}}. \tag{4}$$

The gradient operator can be applied to any tensor. Therefore, denoting a certain tensor quantity by $\mathbb{F}(x', x^3)$, by the definition of the gradient [7–9] and by (4), we have [1–4]

$$\text{grad}\mathbb{F} = \nabla\mathbb{F} = \mathbf{r}^p \partial_p \mathbb{F} = \mathbf{r}^P \partial_P \mathbb{F} + \mathbf{r}^3 \partial_3 \mathbb{F} = \mathbf{r}^{\bar{M}} g^{P}_{\overset{-}{M}} (\partial_P - g^{\bar{3}}_{P} \partial_3)\mathbb{F} + \mathbf{r}^{\bar{3}} \partial_3 \mathbb{F}.$$

Whence, introducing the differential operator

$$N_p = \partial_p - g^{\bar{3}}_{p} \partial_3, \quad \mathbf{N} = \mathbf{r}^p N_p = \mathbf{r}^P N_P = \mathbf{r}^{\bar{M}} g^{P}_{\overset{-}{M}} N_P, \quad N_3 = 0, \tag{5}$$

we obtain the desired representation of the gradient in the form

$$\text{grad}\mathbb{F} = \nabla\mathbb{F} = \mathbf{N}\mathbb{F} + \mathbf{r}^{\bar{3}} \partial_3 \mathbb{F} = \mathbf{r}^P N_P \mathbb{F} + \mathbf{r}^{\bar{3}} \partial_3 \mathbb{F} = \mathbf{r}^{\bar{M}} g^{P}_{\overset{-}{M}} N_P \mathbb{F} + \mathbf{r}^{\bar{3}} \partial_3 \mathbb{F}. \tag{6}$$

The divergence operator is applied to a tensor whose rank is no less than 1. Applying this operator, e.g. to a second-rank tensor $\underset{\sim}{\mathbf{P}}$, by the third relation in (2) and (5), we obtain

$$\text{div}\, \underset{\sim}{\mathbf{P}} = \nabla \cdot \underset{\sim}{\mathbf{P}} = g^{P}_{\overset{-}{M}} N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} \quad (\mathbf{P}^{\bar{m}} = \mathbf{r}^{\bar{m}} \cdot \underset{\sim}{\mathbf{P}}). \tag{7}$$

Note that (7) can also be easily obtained from (6) if in this relation we replace the sign of tensor product, which is omitted, with the sign of inner product.

1.1.1. Representations of equations of motion

As is known [10–13], at very small displacements and rotations and gradients of displacements and rotations three-dimensional equations of motion of micropolar deformable rigid bodies are represented in the form

$$\nabla \cdot \underset{\sim}{\mathbf{P}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad \nabla \cdot \underset{\sim}{\boldsymbol{\mu}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}. \tag{8}$$

Here $\underset{\sim}{\mathbf{P}}$ and $\underset{\sim}{\boldsymbol{\mu}}$ are tensors of stresses and couple stresses, $\underset{\approx}{\mathbf{C}}$ is the discriminant tensor (third-rank tensor) [7], $\mathbf{u}$ is the vector of displacements, $\boldsymbol{\varphi}$ is the vector of (inner) rotation, ρ is the material density, $\mathbf{F}$ is the mass force density, $\mathbf{m}$ is the mass moment density, and $\overset{2}{\otimes}$ is the inner 2-product (for example, $\underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} = \mathbf{r}^i C_{ijk} P^{jk}$). The definition of inner r-product and the problems related to it are considered in [1–7,14]. Proceeding analogously to [15], for the equations of the classical deformable rigid body mechanics (DRBM) under the classical parametrization of thin body domain, in the case considered, from (8), we find the following form of representation of equations of micropolar DRMB:

$$
\begin{gathered}
(1/\sqrt{\overset{(-)}{g}})\partial_P(\sqrt{\overset{(-)}{g}}\,\overset{(-)}{\vartheta}\,\mathbf{P}^P) + \partial_3(\overset{(-)}{\vartheta}\,\mathbf{P}^3) + \rho\,\overset{(-)}{\vartheta}\,\mathbf{F} = \rho\,\overset{(-)}{\vartheta}\,\partial_t^2\mathbf{u},\\
(1/\sqrt{\overset{(-)}{g}})\partial_P(\sqrt{\overset{(-)}{g}}\,\overset{(-)}{\vartheta}\,\boldsymbol{\mu}^P) + \partial_3(\overset{(-)}{\vartheta}\,\boldsymbol{\mu}^3) + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{P}}) + \rho\,\overset{(-)}{\vartheta}\,\mathbf{m} = \overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi},\\
\overset{(-)}{g} = \det(g_{\underset{m}{-}\underset{n}{-}}), \quad g_{\underset{m}{-}\underset{n}{-}} = \mathbf{r}_{\underset{m}{-}} \cdot \mathbf{r}_{\underset{n}{-}}.
\end{gathered}
\tag{9}
$$

It is easy to see that by (7), Eqs. (8) can be rewritten in the form

$$
g^P_{\underset{M}{-}} N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} + \rho\mathbf{F} = \rho\partial_t^2\mathbf{u}, \quad g^P_{\underset{M}{-}} N_P \boldsymbol{\mu}^{\bar{M}} + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho\mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi}. \tag{10}
$$

Note that the following relations hold:

$$
g^P_{\underset{M}{-}} N_P \mathbf{P}^{\bar{M}} = g^P_{\underset{m}{-}} N_P \mathbf{P}^{\bar{m}} = N_P(g^P_{\underset{m}{-}} \mathbf{P}^{\bar{m}}) = N_P(g^P_{\underset{M}{-}} \mathbf{P}^{\bar{M}}) = N_P \mathbf{P}^P;
$$

using them, we can represent Eqs. (10) in the form

$$
\begin{gathered}
N_P \mathbf{P}^P + \partial_3 \mathbf{P}^{\bar{3}} + \rho\mathbf{F} = \rho\partial_t^2\mathbf{u}, \quad N_P \boldsymbol{\mu}^P + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho\mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi},\\
N_P(g^P_{\underset{M}{-}} \mathbf{P}^{\bar{M}}) + \partial_3 \mathbf{P}^{\bar{3}} + \rho\mathbf{F} = \rho\partial_t^2\mathbf{u}, \quad N_P(g^P_{\underset{M}{-}} \boldsymbol{\mu}^{\bar{M}}) + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho\mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi}.
\end{gathered}
\tag{11}
$$

Multiplying each relation in (10) by $\overset{(-)}{\vartheta}$, with the help of the first relation in (2), we have

$$
\begin{gathered}
A^P_{\underset{M}{-}} N_P \mathbf{P}^{\bar{M}} + \overset{(-)}{\vartheta}\,\partial_3 \mathbf{P}^{\bar{3}} + \rho\,\overset{(-)}{\vartheta}\,\mathbf{F} = \rho\,\overset{(-)}{\vartheta}\,\partial_t^2\mathbf{u},\\
A^P_{\underset{M}{-}} N_P \boldsymbol{\mu}^{\bar{M}} + \overset{(-)}{\vartheta}\,\partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{P}}) + \rho\,\overset{(-)}{\vartheta}\,\mathbf{m} = \overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi}.
\end{gathered}
\tag{12}
$$

Note that (9) – (12) are different forms of representation of the equations of micropolar DRBM (8) for the parametrization of thin body domain considered. They are called the different forms of the equations of micropolar deformable rigid thin body mechanic (DRTBM) under the new parametrization of thin body domain. Taking into account the first relation in (3), we can write Eqs. (10) in the form

$$
\begin{gathered}
\sum_{s=0}^{\infty} A^{\bar{P}}_{\underset{M}{+}} (x^3)^s N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} + \rho\mathbf{F} = \rho\partial_t^2\mathbf{u},\\
\sum_{s=0}^{\infty} A^{\bar{P}}_{\underset{M}{+}} (x^3)^s N_P \boldsymbol{\mu}^{\bar{M}} + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho\mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2\boldsymbol{\varphi}.
\end{gathered}
\tag{13}
$$

It is seen that Eqs. (13) contain infinitely many summands. Therefore, they cannot be used in practice. Naturally, we need to consider approximate equations with finitely many summands. In this connection, let us introduce the following definition.

Definition 1.1 *The equations, which are obtained from* (10) *if, in the expansion of* $g^{P}_{\bar{M}}$ *(see the first formula in* (3)*), we preserve first* $s+1$ *terms, are called the equations of approximation of order* s.

Obviously, the equation of approximation of order s are represented in the form

$$\underset{(s)}{g}{}^{P}_{\bar{M}} N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} + \rho\mathbf{F} = \rho\partial_t^2\mathbf{u}, \quad \underset{(s)}{g}{}^{P}_{\bar{M}} N_P \boldsymbol{\mu}^{\bar{M}} + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho\mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}, \tag{14}$$

$$\underset{(s)}{g}{}^{P}_{\bar{M}} = \sum_{m=0}^{s} A^{\bar{P}}_{\overset{+}{M}} (x^3)^m. \tag{15}$$

From (14) for $s=0$, we obtain the equations of zero approximation, for $s=1$ the equations of first approximation, etc.

1.1.2. Representation of the equation of heat influx in micropolar DRTBM

In the general case, the heat influx equation in micropolar DRBM has the form [16]

$$-\nabla\cdot\mathbf{q} + \rho q - T\frac{d}{dt}(\underset{\sim}{\mathbf{a}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \underset{\sim}{\mathbf{d}} \overset{2}{\otimes} \underset{\sim}{\boldsymbol{\mu}}) + W^* = \rho c_p \partial_t T, \tag{16}$$

where $\mathbf{q}$ is the vector of exterior heat influx, q is the mass heat influx, T is the temperature, $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{d}}$ are the tensors of heat extension, $\underset{\sim}{\mathbf{P}} \neq \underset{\sim}{\mathbf{P}}^T$ is the stress tensor, $\underset{\sim}{\boldsymbol{\mu}} \neq \underset{\sim}{\boldsymbol{\mu}}^T$ is the couple stress tensor, W^* is the scattering function, ρ is the medium density, and c_p is the heat capacity under a constant pressure. If we consider the physically linear medium, then the nonlinearity in (16) is in the third summand of the left-hand side. A similar situation holds in a particular variant of this equation, which is obtained from (16) for $\underset{\sim}{\mathbf{d}} = 0$ (see [17]). In the latter case, since both heat capacities c_p and c_v (the heat capacity under a constant volume) cannot be constant simultaneously (independent of the temperature), very often, one assumes that in this summand, the temperature T is replaced with the temperature $T_0 = \text{const}$. Taking into account this assumption, we see that the desired representation of the heat influx equation has the following form analogous to (10):

$$-g^{P}_{\bar{M}} N_P q^{\bar{M}} - \partial_3 q^{\bar{3}} + \rho q - T_0\frac{d}{dt}(\underset{\sim}{\mathbf{a}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \underset{\sim}{\mathbf{d}} \overset{2}{\otimes} \underset{\sim}{\boldsymbol{\mu}}) + W^* = \rho c_p \partial_t T. \tag{17}$$

If necessary, it is easy to write the relations analogous to (9) and (12). Therefore, for brevity, we do not dwell on this. Note that by (17), analogously to (14), the heat influx equation of approximation of order s is represented in the form

$$-\underset{(s)}{g}{}^{P}_{\bar{M}} N_P q^{\bar{M}} - \partial_3 q^{\bar{3}} + \rho q - T_0\frac{d}{dt}(\underset{\sim}{\mathbf{a}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \underset{\sim}{\mathbf{d}} \overset{2}{\otimes} \underset{\sim}{\boldsymbol{\mu}}) + W^* = \rho c_p \partial_t T. \tag{18}$$

1.1.3. Representations of constitutive relations of physical and heat content

In linear micropolar elasticity theory, the constitutive relations of physical content under non-isothermal processes can be represented in the following form by the generalized Duhamel–Neumann principle [16, 17]:

$$\underset{\sim}{\mathbf{P}} = \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\gamma}} - \underset{\sim}{\mathbf{a}}\vartheta) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\varkappa}} - \underset{\sim}{\mathbf{d}}\vartheta), \quad \underset{\sim}{\boldsymbol{\mu}} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\gamma}} - \underset{\sim}{\mathbf{a}}\vartheta) + \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\varkappa}} - \underset{\sim}{\mathbf{d}}\vartheta), \tag{19}$$

where $\underset{\sim}{\boldsymbol{\gamma}} = \nabla \mathbf{u} - \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi}$ is the deformation tensor in micropolar theory (see [12]), $\underset{\sim}{\boldsymbol{\varkappa}} = \nabla \boldsymbol{\varphi}$ is the bend-torsion tensor, $\underset{\approx}{\mathbf{A}}$, $\underset{\approx}{\mathbf{B}}$, $\underset{\approx}{\mathbf{D}}$ ($\underset{\approx}{\mathbf{C}} = \underset{\approx}{\mathbf{B}}^{T}$) are material tensors of the fourth rank, and ϑ is the temperature overfall. By the expression for $\underset{\sim}{\boldsymbol{\gamma}}$, we can write (19) in the form

$$\underset{\sim}{\mathbf{P}} = \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \nabla \mathbf{u} + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \nabla \boldsymbol{\varphi} - \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta, \quad \underset{\sim}{\boldsymbol{\mu}} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \nabla \mathbf{u} + \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} \nabla \boldsymbol{\varphi} - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta, \tag{20}$$

$$\underset{\sim}{\mathbf{b}} = \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \underset{\sim}{\mathbf{a}} + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \underset{\sim}{\mathbf{d}}, \quad \underset{\sim}{\boldsymbol{\beta}} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{a}} + \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} \underset{\sim}{\mathbf{d}},$$

which are called the tensors of thermomechanical properties. Note that a particular case of law (20) was considered in [12, 13], and more general relations were presented in [16, 18]. Now it is easy to find the desired representations of the Hooke law (20) under the new parametrization of thin body domain. Indeed, taking into account the representation of the gradient operator (6), after simple transformations, from (20), we have

$$\begin{aligned} \underset{\sim}{\mathbf{P}} &= \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (g^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \mathbf{u} + \mathbf{r}^{\bar{3}} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (g^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar{3}} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta, \\ \underset{\sim}{\boldsymbol{\mu}} &= \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (g^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \mathbf{u} + \mathbf{r}^{\bar{3}} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (g^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar{3}} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta. \end{aligned} \tag{21}$$

Taking into account the first relation in (3), it is easy to note that relations (21) contain infinitely many summands. Therefore, they cannot be used in such a form. In applications, one uses approximate constitutive relations (CR), i.e., the relations represented by finitely many summands. In this connection, we introduce the following definition.

Definition 1.2 *The relations obtained from* (21) *under the condition that in the expansion of* $g^{P}_{\overline{M}}$ *(see the first formula in* (3)*), first* $s+1$ *are preserved are called the CR of approximation of order* s.

It is easy to see that analogously to Eqs. (14) and (18), CR of approximation of order s are represented in the form

$$\begin{aligned} \underset{\sim}{\mathbf{P}}_{(s)} &= \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \mathbf{u} + \mathbf{r}^{\bar{3}} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar{3}} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta, \\ \underset{\sim}{\boldsymbol{\mu}}_{(s)} &= \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \mathbf{u} + \mathbf{r}^{\bar{3}} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\overline{M}} \mathbf{r}^{\bar{M}} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar{3}} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta. \end{aligned} \tag{22}$$

Definition 1.3 *The relations obtained from* (22) *for* $s = 0$ *are called CR of zero approximation, and for* $s = 1$*, they are called CR of the first approximation.*

It is easy to see that CR of zero approximation have the form

$$\underset{\sim}{\mathbf{P}}_{(0)} = \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (\mathbf{r}^{\bar{M}} N_P \mathbf{u} + \mathbf{r}^{\bar{3}} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (\mathbf{r}^{\bar{M}} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar{3}} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}} \vartheta,$$

$$\underset{\sim}{\boldsymbol{\mu}}_{(0)} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\mathbf{r}^{\bar{M}} N_P \mathbf{u} + \mathbf{r}^{\bar{3}} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (\mathbf{r}^{\bar{M}} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar{3}} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}} \vartheta.$$

Note that if we consider a body without center of symmetry [12,13], then $\underset{\approx}{\mathbf{A}} = 0$, $\underset{\approx}{\mathbf{B}} = 0$, and in this case, the CR presented above simplify. Let us find the corresponding representation for the Fourier heat conduction law (which defines the relations of heat content) under the new parametrization of the thin body domain.Since the Fourier heat conduction law [13,17] has the form $\mathbf{q} = -\underset{\sim}{\Lambda} \cdot \nabla T$, where the second-rank positive-definite tensor $\underset{\sim}{\boldsymbol{\Lambda}}$ is called the heat conduction tensor, by (6), the Fourier heat conduction law of zero approximation and approximation of order s is represented in the form

$$\mathbf{q}_{(0)} = -\boldsymbol{\Lambda}^{\bar{M}} N_P T - \boldsymbol{\Lambda}^{\bar{3}} \partial_3 T, \quad \mathbf{q}_{(s)} = -\boldsymbol{\Lambda}^{\bar{M}} \underset{(s)M}{g}{}^{\underline{P}} N_P T - \boldsymbol{\Lambda}^{\bar{3}} \partial_3 T, \quad \boldsymbol{\Lambda}^{\bar{m}} = \underset{\sim}{\boldsymbol{\Lambda}} \cdot \mathbf{r}^{\bar{m}}. \tag{23}$$

2. To micropolar theory with respect to system of orthonormal Chebyshev polynomials of second kind

To construct the micropolar theory with respect to a certain system of orthogonal polynomials (Legendre, Chebyshev, etc.), we need recursive relations for these polynomials. For example, for the shifted Chebyshev polynomials, the main recursive relations on the orthogonality closed interval [0,1] are represented in the following form [1–4, 19, 20]:

$$\begin{aligned} &4tU_n^*(t) = U_{n-1}^*(t) + 2U_n^*(t) + U_{n+1}^*(t),\ 2tU_n^{*'}(t) = 2nU_n^*(t) + U_{n-1}^{*'}(t) + U_n^{*'}(t),\ n \geq 1, \\ &U_n^{*'}(t) = 4nU_{n-1}^*(t) + U_{n-2}^{*'}(t), \quad n \geq 2, \quad 0 \leq t \leq 1. \end{aligned} \tag{24}$$

Note that formulas (24) are obtained in the same way as analogous formulas for Legendre polynomials on the orthogonality closed interval [-1,1], e.g, in [21] (see also [22]). Using the main recursive relations (24), it is easy to obtain the following relations, which are necessary for constructing thin body theories [1–4, 19, 20]:

$$2^{2s} t^s U_n^*(t) = \sum_{p=0}^{2s} C_{2s}^p U_{n-s+p}^*(t), \quad s, n \in \mathbb{N}_0; \tag{25}$$

$$2^{2s} t^s U_m^*(t) U_n^*(t) = \sum_{p=0}^{m} \sum_{q=0}^{2s} C_{2s}^q U_{n-m-s+2p+q}^*(t), \quad n, m, s \in \mathbb{N}_0; \tag{26}$$

$$U_n^{*'}(t) = 4\sum_{k=0}^{[(n-1)/2]}(n-2k)U^*_{n-(2k+1)}(t) = 4\sum_{k=0}^{[(n-1)/2]}(2k+1+a)U^*_{2k+a}(t),\ n \geq 1; \quad (27)$$

$$2^{2s}t^sU_n^{*'}(t) = 4\sum_{k=0}^{[(n-1)/2]}(2k+1+a)U^*_{2k+a-s+p},\ n\geq 1,\ s\geq 0; \quad (28)$$

$$U_n^{*''}(t) == 2^2\sum_{k=0}^{[(n-2)/2]}(2k+2-a)[(n+1)^2-(2k+2-a)^2]U^*_{2k+1-a}(t), \quad n\geq 2; \quad (29)$$

$$\begin{aligned} 2^{2s}t^sU_n^{*''}(t) &= 2^4\sum_{k=0}^{[(n-2)/2]}\sum_{p=0}^{2s}(k+1)(n-k)[n-(2k+1)]C^p_{2s}U^*_{n-(s+2k+2)+p} \\ &= 2^2\sum_{k=0}^{[(n-2)/2]}(2k+2-a)[(n+1)^2-(2k+2-a)^2]C^p_{2s}U^*_{2k+1-a-s+p},\ n\geq 2,\ s\geq 0. \end{aligned} \quad (30)$$

Here, $a = n-1-2\big[(n-1)/2\big]$, $[x]$ is the integral part of x, and C^n_m are the binomial coefficients. It should be noted that all relations (25) – (30),which also hold for the system of orthonormal Chebyshev polynomials of the second kind $\{\hat{U}^*_k\}^\infty_{k=0}$, except for (26), can be proved by induction. For a system of orthonormal polynomials (26) is presented in the form

$$2^{2s}t^s\hat{U}^*_m(t)\hat{U}^*_n(t) = \hat{U}^*_0\sum_{p=0}^{m}\sum_{q=0}^{2s}C^q_{2s}\hat{U}^*_{n-m-s+2p+q}(t), \quad n,m,s\in\mathbb{N}_0. \quad (31)$$

Note that extending the definition of system of Chebyshev polynomials of the second kind to the set of negative numbers, we obtain the relation $U^*_{-n} = -U^*_{n-2}$, $n\in\mathbb{N}_0$, under which (25) – (30) were obtained.

Let us consider a certain tensor field $\mathbb{F}(x^1,x^2,x^3)$, which depends on the coordinates x^1,x^2,x^3 of the thin body domain under its new parametrization. For brevity, instead of $\mathbb{F}(x^1,x^2,x^3)$, we write $\mathbb{F}(x',x^3)$, where $x'=(x^1,x^2)$, $x^3\in[0,1]$. Moreover, we assume that the tensor fields considered are sufficiently smooth. For example, $\mathbb{F}(x',x^3)\in C_m(V\cup\partial V)$, $m\geq 1$; V is the domain occupied by the thin body considered and ∂V is its boundary. Then the tensor field $\mathbb{F}(x',x^3)$ can be expanded in a series with respect to the system of shifted Chebyshev polynomials of the second kind $\{\hat{U}^*_k\}^\infty_{k=0}$ with respect to the coordinate $x^3\in[0,1]$ for each fixed point $x'\in\overset{(-)}{S}$ [21]. This expansion is represented in the form

$$\mathbb{F}(x',x^3) = \sum_{k=0}^{\infty}\overset{(k)}{\mathbb{F}}(x')\hat{U}^*_k(x^3), \quad x'\in\overset{(-)}{S}, \quad x^3\in[0,1], \quad (32)$$

where $\overset{(k)}{\mathbb{F}}(x')$ is called the coefficient with number k in the expansion of $\mathbb{F}(x',x^3)$ in the series with respect to the polynomial system $\{\hat{U}^*_k\}^\infty_{k=0}$.

Definition 2.1 *The moment of the kth order of a certain tensor field $\mathbb{F}(x',x^3)$ with respect to the polynomial system $\{\hat{U}^*_k\}^\infty_{k=0}$, which is denoted by $\overset{(k)}{\mathbb{M}}(\mathbb{F})$, is the integral*

$$\overset{(k)}{\mathbb{M}}(\mathbb{F}) = \int_0^1\mathbb{F}(x',x^3)\hat{U}^*_k(x^3)h^*(x^3)dx^3, \quad k\in\mathbb{N}_0. \quad (33)$$

It is easy to prove that the following assertions hold:

Assertion 2.1 *For any tensor fields* $\mathbb{F}(x',x^3)$ *and* $\mathbb{G}(x',x^3)$ *and any functions* $\alpha(x')$ *and* $\beta(x')$*, the following relation holds:*

$$\overset{(k)}{\mathbb{M}}[\alpha(x')\mathbb{F}+\beta(x')\mathbb{G}]=\alpha(x')\overset{(k)}{\mathbb{M}}(\mathbb{F})+\beta(x')\overset{(k)}{\mathbb{M}}(\mathbb{G}),\quad k\in\mathbb{N}_0. \tag{34}$$

This implies that the moment operator is a linear operator.

Assertion 2.2 *The kth-order moment of a tensor field* $\mathbb{F}(x',x^3)$ *with respect to the polynomial system* $\{\hat{U}^*_k\}_{k=0}^{\infty}$ *is equal to the coefficient with number k in the expansion of* $\mathbb{F}(x',x^3)$ *with respect to* x^3 *in this polynomial system, i.e.,*

$$\overset{(k)}{\mathbb{M}}(\mathbb{F})=\int\limits_0^1\mathbb{F}(x',x^3)\hat{U}^*_k(x^3)h^*(x^3)dx^3=\overset{(k)}{\mathbb{F}}(x'),\quad k\in\mathbb{N}_0. \tag{35}$$

Assertion (34) follows from (33), whereas (35) is proved by using (32), (33) and the orthonormality of the system $\{\hat{U}^*_k\}_{k=0}^{\infty}$. It is easy to prove that the relations hold

$$\overset{(k)}{\mathbb{M}}(\partial_i\mathbb{F})=\begin{cases}\partial_I\overset{(k)}{\mathbb{F}}(x'), & i=I,\\ \overset{(k)}{\mathbb{F}}{}'(x'), & i=3,\end{cases}\qquad \overset{(k)}{\mathbb{M}}(\partial_i\partial_j\mathbb{F})=\begin{cases}\partial_I\partial_J\overset{(k)}{\mathbb{F}}(x'), & i=I,\ j=J,\\ \partial_I\overset{(k)}{\mathbb{F}}{}'(x'), & i=I,\ j=3,\\ \overset{(k)}{\mathbb{F}}{}''(x'), & i=j=3,\end{cases} \tag{36}$$

where we have introduced the following notation:

$$\begin{aligned}\overset{(k)}{\mathbb{F}}{}'(x')&=2(k+1)\left[\sum_{p=k}^{N}\big(1-(-1)^{k+p}\big)\overset{(p)}{\mathbb{F}}(x')+\overset{(+)}{\mathbb{F}}{}'(x')-(-1)^k\overset{(-)}{\mathbb{F}}{}'(x')\right],\\ \overset{(k)}{\mathbb{F}}{}''(x')&=2(k+1)\left[\sum_{p=k}^{N}(p-k)(k+p+2)\big(1+(-1)^{k+p}\big)\overset{(p)}{\mathbb{F}}(x')+\overset{(+)}{\mathbb{F}}{}''+(-1)^k\overset{(-)}{\mathbb{F}}{}''\right],\end{aligned} \tag{37}$$

$$\begin{aligned}&\overset{(+)}{\mathbb{F}}{}'(x')=\sum_{p=N+1}^{\infty}\overset{(p)}{\mathbb{F}}(x'),\quad \overset{(+)}{\mathbb{F}}{}''(x')=\sum_{p=N+1}^{\infty}(p-k)(k+p+2)\overset{(p)}{\mathbb{F}}(x'),\\ &\overset{(-)}{\mathbb{F}}{}'(x')=\sum_{p=N+1}^{\infty}(-1)^p\overset{(p)}{\mathbb{F}}(x'),\quad \overset{(-)}{\mathbb{F}}{}''(x')=\sum_{p=N+1}^{\infty}(-1)^p(p-k)(k+p+2)\overset{(p)}{\mathbb{F}}(x').\end{aligned} \tag{38}$$

Note that the first relation in (37) can be taken as the definition of the "prime" operator, and the second can be obtained by applying the "prime" operator to $\overset{(k)}{\mathbb{F}}$ two times. The following relations are generalizations of (36):

$$\overset{(k)}{\mathbb{M}}[P_N(x^3)\partial_i^p\partial_j^q\mathbb{F}]=\left[\begin{array}{ll}\partial_I^p\partial_J^q\overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big], & i=I,\ j=J,\\ \partial_I\big\{\overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big]\big\}^{(q)}, & i=I,\ j=3,\\ \big\{\overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big]\big\}^{(p+q)}, & i=j=3.\end{array}\right. \tag{39}$$

where $P_N(x^3)$ is a polynomial of degree N, k, N, p, $q \in \mathbb{N}_0$, and $\{\overset{(k)}{\mathbb{M}}[P_N(x^3)\mathbb{F}]\}^{(m)}$, $m \in \mathbb{N}_0$, means that the "prime" operator is applied m times. To prove first lines of (36) and (39), we use Definition (33). The second and third lines of (36) are proved by using (27) and (29), respectively, and the second and third lines of (39) are proved by induction. Using (25) and the last relation in (39), we can prove the relations

$$\overset{(n)}{\mathbb{M}}[(x^3)^s\mathbb{F}]=\sum_{p=0}^{2s} 2^{-2s}C_{2s}^p \overset{(n-s+p)}{\mathbb{F}}, \quad \overset{(n)}{\mathbb{M}}[(x^3)^s\partial_3^m\mathbb{F}]=\sum_{p=0}^{2s} 2^{-2s}C_{2s}^p \overset{(n-s+p)}{\mathbb{F}}{}^{(m)}, \quad n,\ s,\ m\in\mathbb{N}_0. \tag{40}$$

It can be seen that the first relation (40) is obtaind from the second for $m = 0$. Let us represent (40) for $m = 1$ in another form. Using easy transformations, the first relation in (37) and first two relations in (38), from the second relation (40) for $m = 1$, we obtain

$$\begin{aligned}
&\overset{(k)}{\mathbb{M}}{}'[(x^3)^{s+1}\mathbb{F}] = \overset{(k)}{\mathbb{M}}[(x^3)^{s+1}\partial_3\mathbb{F}] = \sum_{p=0}^{2s+2} 2^{-2(s+1)}C_{2s+2}^p \overset{(p-v)}{\mathbb{F}}{}' \\
&= \sum_{p=0}^{2s+2}\sum_{q=l-1}^{N} l\, 2^{-2(s+1)}C_{2s+2}^p\big[1-(-1)^{l+q}\big]\overset{(q)}{\mathbb{F}} + a_{(s,k)}\overset{(+)}{\mathbb{F}}{}', \\
&v = s+1-k,\quad l = k-s+p,\quad s\geq 0,\quad k\geq 0.
\end{aligned} \tag{41}$$

where we have introduced the notation $a_{(s,k)}= 2^{-(2s+1)}\sum_{p=0}^{2s+2}(k-s+p)C_{2s+2}^p$, $s \geq 0$, $k \geq 0$.

Let us find the expression for $\overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{\underset{-}{P}}\mathrm{N}_P\mathbb{F})$. By (5) and (34), we obtain

$$\overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{\underset{-}{P}}\mathrm{N}_P\mathbb{F}) = \overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{\underset{-}{P}}\partial_P\mathbb{F}) - g^{\bar{3}}_{\underset{P}{+}}\overset{(k)}{\mathbb{M}}(x^3\underset{(s)M}{g}{}^{\underset{-}{P}}\partial_3\mathbb{F}). \tag{42}$$

Furthermore, by (15), (34), (39) and the first relation (40), we find that

$$\overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{\underset{-}{P}}\partial_P\mathbb{F}) = \sum_{m=0}^{s}\sum_{n=0}^{2m} 2^{-2m}C_{2m}^n \underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\partial_P \overset{(k-m+n)}{\mathbb{F}}, \quad k,\ s \in \mathbb{N}_0 \tag{43}$$

Whence, for $s = 0$ and $s = 1$, we obtain

$$\overset{(k)}{\mathbb{M}}(\underset{(0)M}{g}{}^{\underset{-}{P}}\partial_P\mathbb{F})=\partial_M\overset{(k)}{\mathbb{F}}, \quad \overset{(k)}{\mathbb{M}}(\underset{(1)M}{g}{}^{\underset{-}{P}}\partial_P\mathbb{F})=\partial_P\overset{(k)}{\mathbb{F}}+\frac{1}{4}A^{\bar{P}}_{\underset{M}{+}}\partial_M(\overset{(k-1)}{\mathbb{F}}+2\overset{(k)}{\mathbb{F}}+\overset{(k+1)}{\mathbb{F}}), \quad k\geq 0. \tag{44}$$

Here, we have introduced the notation $A^{\bar{P}}_{\underset{M}{+}} \equiv \underset{(1)}{A}{}^{\bar{P}}_{\underset{M}{+}} = g^{\bar{P}}_{\underset{M}{-}} - g^{\bar{P}}_{\underset{M}{+}}$. Moreover, we assume that $\overset{(m)}{\mathbb{F}} = 0$ if $m < 0$. On what follows, we assume that this condition holds. Analogously, by (15), (34), (39) and (41), we have

$$\begin{aligned}
&\overset{(k)}{\mathbb{M}}(x^3\underset{(s)M}{g}{}^{\underset{-}{P}}\partial_3\mathbb{F}) = \sum_{m=0}^{s}\sum_{n=0}^{2m+2} 2^{-2(m+1)}C_{2m+2}^n \underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\overset{(n-v)}{\mathbb{F}}{}'(x') \\
&= \sum_{m=0}^{s}\sum_{n=0}^{2m+2}\sum_{q=l-1}^{N} 2^{-2(m+1)}C_{2m+2}^n l\, \underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\big[1+(-1)^{l+q}\big]\overset{(q)}{\mathbb{F}}(x')+\Big(\sum_{m=0}^{s} a_{(m,k)}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\Big)\overset{(+)}{\mathbb{F}}{}', \\
&v = m+1-k,\quad l\equiv k-m+n,\quad N\geq s+k+1,\quad k\geq 0,\quad s\geq 0.
\end{aligned} \tag{45}$$

Whence, for $s = 0$ and $s = 1$, we find that

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}(x^3 \underset{(0)}{g}{}^{P}_{-M}\partial_3\mathbb{F}) = g^{\bar{P}}_{-M}\overset{(k)}{\mathbb{M}}{}'(x^3\mathbb{F}) = \frac{1}{4} g^{\bar{P}}_{-M}\big(\overset{(k-1)}{\mathbb{F}}{}' + 2\overset{(k)}{\mathbb{F}}{}' + \overset{(k+1)}{\mathbb{F}}{}'\big)\\
&= g^{\bar{P}}_{-M}\big[k\overset{(k)}{\mathbb{F}} + 2(k+1)\big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\big)\big], \quad k \geq 0,
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}(x^3 \underset{(1)}{g}{}^{P}_{-M}\partial_3\mathbb{F}) = \overset{(k)}{\mathbb{M}}\big[\big(g^{\bar{P}}_{-M}x^3 + A^{\bar{P}}_{+M}(x^3)^2\big)\partial_3\mathbb{F}\big] = g^{\bar{P}}_{-M}\overset{(k)}{\mathbb{M}}{}'(x^3\mathbb{F}) + A^{\bar{P}}_{+M}\overset{(k)}{\mathbb{M}}{}'[(x^3)^2\mathbb{F}]\\
&= g^{\bar{P}}_{-M}\big[k\overset{(k)}{\mathbb{F}} + 2(k+1)\big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\big)\big] + \frac{1}{4}A^{\bar{P}}_{+M}\big[(k-1)\overset{(k-1)}{\mathbb{F}}\\
&-4(k+2)\overset{(k)}{\mathbb{F}} - (k+3)\overset{(k+1)}{\mathbb{F}} + 8(k+1)\big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\big)\big], \quad k \geq 0.
\end{aligned}
\tag{47}
$$

Taking into account (43) and (45), from (42) we obtain the desired relation in the form

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}(\underset{(r)}{g}{}^{P}_{-M}N_P\mathbb{F}) = \sum_{s=0}^{r}\underset{(s)}{A}{}^{\bar{P}}_{+M}\big\{\nabla_P\overset{(k)}{\mathbb{M}}[(x^3)^s\mathbb{F}] - g^{\bar{3}}_{+P}\overset{(k)}{\mathbb{M}}{}'[(x^3)^{s+1}\mathbb{F}]\big\}\\
&= \sum_{s=0}^{r}\sum_{p=0}^{2s}2^{-2s}\underset{(s)}{A}{}^{\bar{P}}_{+M}C^p_{2s}\nabla_P\overset{(p-u)}{\mathbb{F}} - g^{\bar{3}}_{+P}\sum_{s=0}^{r}\sum_{p=0}^{2s+2}2^{-2(s+1)}\underset{(s)}{A}{}^{\bar{P}}_{+M}C^p_{2s+2}\overset{(p-v)}{\mathbb{F}}{}',\\
&u = s-k, \ v = u+1,
\end{aligned}
\tag{48}
$$

where the expression for $\overset{(p-v)}{\mathbb{F}}{}'$ is given by the first formula (37) for $k = p - v$.

From (48) for $s = 0$ and $s = 1$ by (44), (46) and (47) we find that

$$
\overset{(k)}{\mathbb{M}}(\underset{(0)}{g}{}^{J}_{-I}N_J\mathbb{F}) = \overset{(k)}{\mathbb{M}}(N_I\mathbb{F}) = \nabla_I\overset{(k)}{\mathbb{F}} - g^{\bar{3}}_{+I}\big[k\overset{(k)}{\mathbb{F}} + 2(k+1)\big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\big)\big], \ k \geq 0, \tag{49}
$$

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}(\underset{(1)}{g}{}^{J}_{-I}N_J\mathbb{F}) = \overset{(k)}{\mathbb{M}}[(g^{\bar{J}}_{-I} + x^3A^{\bar{J}}_{+I})N_J\mathbb{F}] = \nabla_I\overset{(k)}{\mathbb{F}} + \frac{1}{4}A^{\bar{J}}_{+I}\nabla_J\big(\overset{(k-1)}{\mathbb{F}} + 2\overset{(k)}{\mathbb{F}} + \overset{(k+1)}{\mathbb{F}}\big)\\
&-g^{\bar{3}}_{+J}\big\{g^{\bar{J}}_{-I}\big[k\overset{(k)}{\mathbb{F}} + 2(k+1)\big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\big)\big] + \frac{1}{4}A^{\bar{J}}_{+I}\big[(k-1)\overset{(k-1)}{\mathbb{F}} - 4(k+2)\overset{(k)}{\mathbb{F}}\\
&-(k+3)\overset{(k+1)}{\mathbb{F}} + 8(k+1)\big(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}'\big)\big]\big\}, \quad k \geq 0.
\end{aligned}
\tag{50}
$$

Therefore, we have deduced the main relation in the form (48); using this relation from the equations of motion (14), the heat influx equation (18), CR of physical content (22), and CR of heat content (23) (second formula) of approximation of order r, we obtain the corresponding relations in moments; in turn by using the above rule, from them, we obtain the corresponding relations for multilayer thin bodies. Analogously, we obtain the boundary conditions of physical and heat contents in moments. Formulas (49) and (50) are applied in deducing the above relations from the corresponding relations of zero and first approximations. Formulas analogous to (49) and (50) certainly hold for the Legendre polynomial system. The relation for Legendre polynomial system analogous to (48) is very cumbersome, and so we do not write it.

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