Formulation of the initial boundery velue problems in the theory of multilayer thermoelastic thin bodies in moments (part III)

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Abstract: Various variants of the equations of motion in moments with respect to orthogonal polynomial systems are obtained. The interlayer conditions are written down under various connections of adjacent layers of a multilayer body. Formulation of the initial boundary value problems in the theory of multilayer thermoelastic thin bodies in moments are given.

Note that the analytic method with the use of the Legendre polynomial system in constructing the one-layer thin body theory and multilayer thin body theory can be successfully used in constructing any thin body theory. Despite this, the classic theories constructed by this method are far to be complete, and the more so, the micropolar theories and theories of other rheology are.

1. Systems of equations of motion in moments for multilayer thin bodies with one small size

1.1. Systems of equations of motion in moments of contravariant components of stress tensors and couple stresses with respect with respect to Chebyshev polynomial systems for multilayer thin bodies with one small size

We restrict ourselves to obtaining the systems of equations of motion of approximations $(0, N)$ and $(1, N)$ in moments. Using the rule presented above, by analogous systems of equations from [1, 2], we represent the desired systems of equations in the form

$$
\begin{aligned}
&\Big\{\nabla_I \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} - \underset{\alpha\,I}{g_{+}^{\bar 3}}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} + 2(k+1)\Big(\sum_{p=k}^{N}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar I} - \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I}\Big)\Big]\\
&+2(k+1)\sum_{p=k}^{N}\Big[1-(-1)^{k+p}\Big]\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar 3}\Big\} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{F}}} = \underset{\alpha}{\rho}\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf{u}}},\\
&\Big\{\mathbf{P}\Rightarrow\boldsymbol{\mu}\Big\} + \underset{\approx}{\underset{\alpha}{\mathbf{C}}}\overset{2}{\otimes}\underset{\widetilde{\alpha}}{\overset{(k)}{\mathbf{P}}} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{m}}} = \underset{\widetilde{\alpha}}{\mathbf{J}}\cdot\partial_t^2\underset{\alpha}{\overset{(k)}{\boldsymbol{\varphi}}},\ \ k=\overline{0,N},\ \ \alpha=\overline{0,K};
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&\Bigg\{\nabla_I \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} + \frac{1}{4}\Big(\underset{\alpha\,I}{g_{-}^{\bar J}} - \underset{\alpha\,I}{g_{+}^{\bar J}}\Big)\nabla\nabla_J\Big(\underset{\alpha}{\overset{(k-1)}{\mathbf{P}}}{}^{\bar I} + 2\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} + \underset{\alpha}{\overset{(k+1)}{\mathbf{P}}}{}^{\bar I}\Big) - \underset{\alpha\,I}{g_{+}^{\bar 3}}\Big[\underset{\alpha\,J}{g_{-}^{\bar I}}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar J} + 2(k+1)\Big(\sum_{p=k}^{N}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar J} - \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar J}\Big)\Big]\\
&+\frac{1}{4}\Big(\underset{\alpha\,J}{g_{-}^{\bar I}} - \underset{\alpha\,J}{g_{+}^{\bar I}}\Big)\Big[(k-1)\underset{\alpha}{\overset{(k-1)}{\mathbf{P}}}{}^{\bar J} - 4(k+2)\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar J} - (k+3)\underset{\alpha}{\overset{(k+1)}{\mathbf{P}}}{}^{\bar J}\\
&+8(k+1)\Big(\sum_{p=k}^{N}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar J}\Big)\Big]\Big] + 2(k+1)\Big[\sum_{p=k}^{N}(1-(-1)^{k+p})\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar 3}\Big]\Bigg\} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{F}}} = \underset{\alpha}{\rho}\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf{u}}},
\end{aligned}
\tag{2}
$$

$$
\Big\{\mathbf{P}\Rightarrow\boldsymbol{\mu}\Big\}+\underset{\alpha}{\underset{\approx}{\mathbf{C}}}\overset{2}{\otimes}\underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf{P}}}}+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{m}}}=\underset{\alpha}{\underset{\sim}{\mathbf{J}}}\cdot\partial_t^2\underset{\alpha}{\overset{(k)}{\boldsymbol{\varphi}}},\quad k=\overline{0,N},\quad \alpha=\overline{0,K}.
$$

Here the notation $\{\mathbf{P}\Rightarrow\boldsymbol{\mu}\}$ means that the expression in brackets is obtained from the expression in brackets of the previous relation if the letter $\mathbf{P}$ is replaced with $\boldsymbol{\mu}$; the notation is also used later. Note that Eqs. (1) and (2) are also obtained by using formulas (II: 25) and (II: 27).

1.2. Systems of equations of motion in moments of contravariant components of stress tensors and couple stresses with respect to Legendre polynomial systems for multilayer thin bodies with one small size

Let us write systems of equations of motion of approximations $(0,N)$ and $(1,N)$ in moments taking into account only boundary conditions of physical content on frontal surface, since the systems of equations without boundary conditions on frontal surfaces, which can be obtained by using the corresponding systems of equations from [1, 2], have the form analogous to (1) and (2). It is easy to prove that analogously to the system of equations for a one-layer classical elastic body [3], the desired systems of equations have the form (see also [1, 2])

$$
\begin{aligned}
&\Bigg\{\nabla_I\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I}-\underset{\alpha I}{g_{+}^{\bar 3}}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I}-(2k+1)\sum_{p=0}^{k}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar I}\Big]-(2k+1)\sum_{p=0}^{k}\big[1-(-1)^{k+p}\big]\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar 3}\\
&+(2k+1)\Bigg[\sqrt{\underset{\alpha}{g}{}^{\overset{++}{33}}}\underset{\alpha}{\overset{(+)}{\mathbf{P}}}+(-1)^k\sqrt{\underset{\alpha}{g}{}^{\overset{--}{33}}}\underset{\alpha}{\overset{(-)}{\mathbf{P}}}\Bigg]\Bigg\}+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{F}}}=\underset{\alpha}{\rho}\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf{u}}},
\end{aligned}\tag{3}
$$

$$
\Big\{\mathbf{P}\Rightarrow\boldsymbol{\mu}\Big\}+\underset{\alpha}{\underset{\approx}{\mathbf{C}}}\overset{2}{\otimes}\underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf{P}}}}+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{m}}}=\underset{\alpha}{\underset{\sim}{\mathbf{J}}}\cdot\partial_t^2\underset{\alpha}{\overset{(k)}{\boldsymbol{\varphi}}},\quad k=\overline{0,N},\quad \alpha=\overline{0,K};
$$

$$
\begin{aligned}
&\Bigg\{\nabla_I\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I}+\frac{1}{2}\Big(\underset{\alpha M}{g_{-}^{\bar P}}-\underset{\alpha M}{g_{+}^{\bar P}}\Big)\Big(\frac{k}{2k-1}\nabla_P\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M}+\nabla_P\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M}+\frac{k+1}{2k+3}\nabla_P\underset{\alpha}{\overset{(k+1)}{\mathbf{P}}}{}^{\bar M}\Big)\\
&-(2k+1)\sum_{p=0}^{k}\big[1-(-1)^{k+p}\big]\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar 3}-\underset{\alpha P}{g_{+}^{\bar 3}}\Bigg[\underset{\alpha M}{g_{-}^{\bar P}}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M}-(2k+1)\sum_{p=0}^{k}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar M}\Big]\\
&+\big(\underset{\alpha M}{g_{-}^{\bar P}}-\underset{\alpha M}{g_{+}^{\bar P}}\big)\Big[\frac{(k-1)k}{2(2k-1)}\underset{\alpha}{\overset{(k-1)}{\mathbf{P}}}{}^{\bar M}+k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M}-\frac{(k+1)(k+2)}{2(2k+3)}\underset{\alpha}{\overset{(k+1)}{\mathbf{P}}}{}^{\bar M}\\
&-(2k+1)\sum_{p=0}^{k}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar M}\Big]\Bigg]+(2k+1)\Bigg[\sqrt{\underset{\alpha}{g}{}^{\overset{++}{33}}}\underset{\alpha}{\overset{(+)}{\mathbf{P}}}+(-1)^k\sqrt{\underset{\alpha}{g}{}^{\overset{--}{33}}}\underset{\alpha}{\overset{(-)}{\mathbf{P}}}\Bigg]\Bigg\}+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{F}}}=\underset{\alpha}{\rho}\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf{u}}},
\end{aligned}\tag{4}
$$

$$
\Big\{\mathbf{P}\Rightarrow\boldsymbol{\mu}\Big\}+\underset{\alpha}{\underset{\approx}{\mathbf{C}}}\overset{2}{\otimes}\underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf{P}}}}+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{m}}}=\underset{\alpha}{\underset{\sim}{\mathbf{J}}}\cdot\partial_t^2\underset{\alpha}{\overset{(k)}{\boldsymbol{\varphi}}},\quad k=\overline{0,N},\quad \alpha=\overline{0,K}.
$$

Note that Eqs. (3) and (4) are deduced by using formulas for Legendre polynomials that are analogous to (II: 25) and (II: 27). Also, note that $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}$ ($\underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}}$) and $\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}$ ($\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}$) ($\alpha=\overline{1,K-1}$) are stress vectors (couple-stresses) of interaction between the layers α and $\alpha+1$, which act on the surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha+1}{\overset{(-)}{S}}$, respectively, and $\underset{1}{\overset{(+)}{\mathbf{P}}}$ ($\underset{1}{\overset{(+)}{\boldsymbol{\mu}}}$) and $\underset{K}{\overset{(-)}{\mathbf{P}}}$ ($\underset{K}{\overset{(-)}{\boldsymbol{\mu}}}$) are given stress vectors (couple-stresses) on the frontal surfaces $\underset{1}{\overset{(+)}{S}}$ and $\underset{K}{\overset{(-)}{S}}$, respectively. The systems of equations of heat influx of approximations $(0, N)$ and $(1, N)$, and also CR of heat content for multilayer thin bodies are obtained in full analogy with (1)–(4). Therefore, for brevity, we do not dwell on them. To understand the article we refer [1, 2, 4], where for the theories of one-layer thin body with one small size and two small sizes, and also for theory of multilayer constructions with the use of the Legendre and Chebyshev polynomial systems, many analogous problems are presented in detail. In particular, using the deduced recursive relations for the Legendre and Chebyshev polynomial systems, these works obtained the moments of derivatives of the first and second order of a scalar function, tensors of the first and second ranks and their components, and also some differential operators of these quantities. These works obtained constitutive relations of physical and heat contents, equations of motion and heat influx, boundary conditions of various kind in moments with respect to the Legendre and Chebyshev polynomial systems, and also initial conditions of kinematic and heat contents. Moreover, the constitutive relations were obtained for an inhomogeneous material. These works presented the statements of related and non-related dynamical problems in moments of approximation (r, N) of micropolar thermomechanics of a deformable rigid thin body and also the statement of non-stationary temperature problem in moments of approximation (r, N), where r and N are arbitrary nonnegative integers. All relations for one-layer thin body presented in this paragraph are automatically transferred to the case of multilayer thin body theory by using the rule presented above.

1.3. Systems of equations in moments of the displacement vector with respect to Legendre and Chebyshev polynomial systems for multilayer thin bodies with one small size

Let us write systems of equations of zero and first approximation in moments for the displacement vector. The system of equations of zero approximation in moments with respect to Legendre and Chebyshev polynomials has the form

$$
\begin{aligned}
&\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{I}\cdot\bar{J}\cdot}\cdot\nabla_I\nabla_J\underset{\alpha}{\overset{(k)}{\mathbf{u}}}+\Big(\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{3}\cdot\bar{I}\cdot}+\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{I}\cdot\bar{3}\cdot}\Big)\cdot\nabla_I\underset{\alpha}{\overset{(k)}{\mathbf{u}}}{}'+\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{3}\cdot\bar{3}\cdot}\cdot\underset{\alpha}{\overset{(k)}{\mathbf{u}}}{}''\\
&-\Big(\underset{\alpha}{\mathbf{b}}^{\bar{I}}\nabla_I\underset{\alpha}{\overset{(k)}{\vartheta}}+\underset{\alpha}{\mathbf{b}}^{\bar{3}}\underset{\alpha}{\overset{(k)}{\vartheta}}{}'\Big)+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{F}}}=\underset{\alpha}{\rho}\,\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf{u}}},\quad k\in\mathbb{N}_0,\quad \alpha=\overline{0,K}.
\end{aligned}
\tag{5}
$$

Taking into account the formulas for moments of the kth order of first and second derivatives of a vector(vector components, a scalar function) with respect to these polynomial systems [1, 2], from (5), we obtain the desired systems of equations of zero approximation in moments. For brevity, we do not write them. Analogously to (5), the system of equations of the first approximation is represented in the form

$$
\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{M}\cdot\bar{N}\cdot}\cdot\Bigg\{\underset{\alpha\bar{M}}{g}^{\bar{P}}\,\underset{\alpha\bar{N}}{g}^{\bar{Q}}\nabla_P\nabla_Q\underset{\alpha}{\overset{(k)}{\mathbf{u}}}+\Bigg[\underset{(1\alpha)}{B}{}_{\overset{+}{M}\overset{+}{N}}^{\bar{P}\bar{Q}}\nabla_P\nabla_Q\overset{(k)}{\mathbf{M}}(x^3\underset{\alpha}{\mathbf{u}})+\underset{(2\alpha)}{B}{}_{\overset{+}{M}\overset{+}{N}}^{\bar{P}\bar{Q}}\nabla_P\nabla_Q\overset{(k)}{\mathbf{M}}\big((x^3)^2\underset{\alpha}{\mathbf{u}}\big)\Bigg]\Bigg\}
$$

$$
+\Big(\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{3}\cdot\bar{M}\cdot}+\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{M}\cdot\bar{3}\cdot}\Big)\cdot\Big[g^{\bar{P}}_{\underset{M}{-}}\nabla_P\underset{\alpha}{\overset{(k)}{\mathbf{u}}}{}'+\big(g^{\bar{P}}_{\underset{\alpha M}{-}}-g^{\bar{P}}_{\underset{\alpha M}{+}}\big)\nabla_P\overset{(k)}{\mathbf{M}}{}'\Big(x^3\underset{\alpha}{\mathbf{u}}\Big)\Big]+\underset{\widetilde{\alpha}}{\mathbf{A}}^{\bar{3}\cdot\bar{3}\cdot}\cdot\underset{\alpha}{\overset{(k)}{\mathbf{u}}}{}'' \tag{6}
$$

$$
-\underset{\alpha}{\mathbf{b}}^{\bar{M}}\Big[g^{\bar{P}}_{\underset{\alpha M}{-}}\nabla_P\underset{\alpha}{\overset{(k)}{\vartheta}}+\Big(g^{\bar{P}}_{\underset{\alpha M}{-}}-g^{\bar{P}}_{\underset{\alpha M}{+}}\Big)\nabla_P\overset{(k)}{\mathbf{M}}\big(x^3\underset{\alpha}{\vartheta}\big)\Big]-\underset{\alpha}{\mathbf{b}}^{\bar{3}}\underset{\alpha}{\overset{(k)}{\vartheta}}{}')+\underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{F}}}=\underset{\alpha}{\rho}\,\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf{u}}},\ k\in\mathbb{N}_0,\ \alpha=\overline{0,K},
$$

where we have introduced the following notation:

$$
\underset{(1\alpha)}{B}{}^{\bar{P}\bar{Q}}_{\underset{M}{+}\underset{N}{+}}=\Big(g^{\bar{P}}_{\underset{\alpha M}{-}}-g^{\bar{P}}_{\underset{\alpha M}{+}}\Big)g^{\bar{Q}}_{\underset{\alpha N}{-}}+g^{\bar{P}}_{\underset{\alpha M}{-}}\Big(g^{\bar{Q}}_{\underset{\alpha N}{-}}-g^{\bar{Q}}_{\underset{\alpha N}{+}}\Big),\quad \underset{(2\alpha)}{B}{}^{\bar{P}\bar{Q}}_{\underset{M}{+}\underset{N}{+}}=\underset{(2\alpha)}{A}{}^{\bar{P}}_{\underset{M}{+}}g^{\bar{Q}}_{\underset{\alpha N}{-}}+\underset{(1\alpha)}{A}{}^{\bar{P}}_{\underset{M}{+}}\underset{(1\alpha)}{A}{}^{\bar{Q}}_{\underset{N}{+}}+g^{\bar{P}}_{\underset{\alpha M}{-}}\underset{(2\alpha)}{A}{}^{\bar{Q}}_{\underset{N}{+}},
$$

$$
\underset{(1\alpha)}{A}{}^{\bar{P}}_{\underset{M}{+}}=g^{\bar{P}}_{\underset{\alpha M}{-}}-g^{\bar{P}}_{\underset{\alpha M}{+}},\qquad \underset{(2\alpha)}{A}{}^{\bar{P}}_{\underset{M}{+}}=\Big(g^{\bar{P}}_{\underset{\alpha N}{-}}-g^{\bar{P}}_{\underset{\alpha N}{+}}\Big)\Big(g^{\bar{N}}_{\underset{\alpha M}{-}}-g^{\bar{N}}_{\underset{\alpha M}{+}}\Big).
$$

Taking into account the expressions for the kth-order moments entering (6) and using the corresponding formulas for Legendre and Chebyshev polynomial systems [1, 2] from (6), we obtain various representations of the equations of of first approximation of the displacement vector in moments with respect to these polynomial systems. It is easy to deduce the equations of motion of first approximation in moments for the displacement vector with respect to the systems of Chebyshev polynomials of the first kind. For brevity, we do not write the equations in moments mentioned in this paragraph.

Note that to close systems (1)–(4), we need to add to them the system of equations of heat influx, CR, boundary and initial conditions of physical and heat contents in moments of the corresponding approximations, and also inter-layer contact conditions depending on the connections of neighboring surfaces. Hence to close systems (5) and (6), we need to add to them all relations of the previous proposition, except for CR in the case of first boundary-value problem where kinematic boundary conditions are given on the whole surface. Of course, it is easy to write all missed relations and formulate the statement of problems analogous to those presented in [1, 2, 4, 5, 6] (CR; see also above) for one-layer domains, except for inter-layer contact conditions, by using the rule presented above. Otherwise, we need to repeat almost all presented in [1, 2, 4] being applied to multilayer thin body theory. Owing to this, we do not dwell on them and consider inter-layer contact conditions below.

2. Inter-Layer Contact Conditions

In studying strained-deformed states of multilayer constructions and composite media, as a rule, one assumes that component layers (elements, phases) work jointly, without sliding. Obviously, such a model does not cover the variety of connection methods used in technology and does not take into account the existence of interphase defects, which manifest themselves in non-perfect connection of phases in contact. Defects of such a type often are undoubt because of peculiarities of technological character (see [7, 8, 9]). Therefore, the deformation of multilayer thin bodies can be without violation or with violation of complete layer contact owing to their separation in normal or tangential direction. Between the layers, there can arise contact domain and contact-free domain. Moreover, the boundaries of these domains can vary in the deformation process, the layer can slide with respect to each other, the sliding can be with friction,etc. All these phenomena can essentially influence on the mechanical behavior of a thin body, its strained-deformed state. Of course, the account of these phenomena is necessary in studying strained-deformed state of multilayer bodies. In contrast to other parametrization, the use of frontal surfaces as base surfaces in parametrization of multilayer thin body domain allows one to easily take into account these phenomena. In consideration the phenomena occurring on frontal surfaces, the main problem is the problem of modelling the interface. In this direction,there exist two approaches. The first approach is physical, which take into account thin adhesion layers via generalized weld condition of elements being in contact. For the first time, such an approach was proposed for heat conduction problems in [10]. Later on, it was generalized to mechanical problems [11]. The second approach is phenological; it is based on the assumption that a priori, the exist discontinuity zones of displacements. To study these problems, we assume that a multilayer thin construction consists of K layers. Denote by $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha}{\overset{(-)}{S}}$ $(\alpha = \overline{1,K})$ the exterior and inner surfaces of the layer α $(\alpha = \overline{1,K})$, respectively and consider several cases of mutual relation of neighboring surfaces $\underset{\alpha}{\overset{(+)}{S}}$ $\underset{\alpha+1}{\overset{(-)}{S}}$ $(\alpha = \overline{1,K-1})$, which are important in practice.

2.1. Weld conditions (complete ideal contact conditions)

In this case the forces and moments of interaction between the layers α and $\alpha+1$ $(\alpha = \overline{1,K-1})$ are unknown. These forces and moments certainly are equal and have opposite directions. Therefore, there additionally arise six unknown functions. However, in the case considered, we have six additional conditions, which express the continuity of displacement vectors and the rotation of welded surface points. In other words, displacement vectors and rotation vectors of contacted surfaces are equal. Denoting the forces and moments of interaction of the contacted surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha+1}{\overset{(-)}{S}}$ $(\alpha = \overline{1,K-1})$ by $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}$, $\underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}}$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}$, $\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}$ $(\alpha = \overline{1,K-1})$, respectively, and the displacement and rotation vectors of points of these surfaces by $\underset{\alpha}{\overset{(+)}{\mathbf{u}}}$, $\underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}}$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}$, $\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}$ $(\alpha = \overline{1,K-1})$, we can represent the complete contact conditions in micropolar theory of multilayer thin bodies in the form

$$\underset{\alpha}{\overset{(+)}{\mathbf{P}}} = -\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}, \quad \underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}} = -\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}, \quad \underset{\alpha}{\overset{(+)}{\mathbf{u}}} = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}, \quad \underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}} = \underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}, \quad \alpha = \overline{1,K-1}. \tag{7}$$

Neglecting the characteristics of micropolar theory (the second and fourth relations) in (7), we obtain the ideal contact conditions for the classical theory (the first and third relations).

2.2. Conditions under relative displacement of contacted layer surfaces

As above, in the process of deformation of a multilayer construction, relative displacements of points of the surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha+1}{\overset{(-)}{S}}$ withe the same Gaussian coordinates (x^1, x^2) are possible. Let us consider various variants. First, we note that there exist bounded limit intensities of coupling forces of the layers α and $\alpha+1$ ($\alpha = \overline{1, K-1}$) in normal and tangential direction. Denote the normal and tangent components of the limit force of action of the layer α on the layer $\alpha+1$ by

$$\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}^{*}_{(n)} = \underset{\alpha+1}{\overset{(-)}{P}}{}^{*}_{(n)}(x^1, x^2)\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}^{*}_{(s)} = \underset{\alpha+1}{\overset{(-)}{P}}{}^{*}_{(s)}(x^1, x^2, \underset{\alpha+1}{\overset{(-)}{\mathbf{s}}})\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}, \quad \alpha = \overline{1, K-1},$$

respectively. Here, certainly, $\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}$ are unit exterior normal and tangent vectors to the surface $\underset{\alpha+1}{\overset{(-)}{S}}$. Note that we take into account the possibility of dependence of the limit tangent force preventing the mutual sliding of layers of the direction in the tangent plane (anisotropy of the limit tangent force).

2.3. Conditions under relative displacement of points of ideal (smooth) contacted layer surfaces

In this case, a free slipping of layers with respect to each other can take place in the process of deformation of a multilayer thin body. The parametrization retains valid all the relations of the theory of thin bodies in the case considered here, only the required and given functions are changed. Obviously, if the layers are united, then the following equalities hold:

$$\begin{gathered}\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha}{\overset{(\mathring{+})}{\mathbf{r}}}(x^1, x^2) + \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1, x^2), \ \ \underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha}{\overset{(\mathring{-})}{\mathbf{r}}}(x^1, x^2) + \underset{\alpha}{\overset{(-)}{\mathbf{u}}}(x^1, x^2), \ \ \alpha = \overline{1, K}, \\ \underset{\alpha}{\overset{(\mathring{+})}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha+1}{\overset{(\mathring{-})}{\mathbf{r}}}(x^1, x^2), \ \ \underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha+1}{\overset{(-)}{\mathbf{r}}}(x^1, x^2), \ \ (\underset{\alpha}{\overset{(+)}{\mathbf{u}}} = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}), \ \ \alpha = \overline{1, K-1}.\end{gathered} \tag{8}$$

where $\underset{\alpha}{\overset{(+)}{\mathbf{r}}}$ $(\underset{\alpha}{\overset{(\mathring{+})}{\mathbf{r}}})$ and $\underset{\alpha}{\overset{(-)}{\mathbf{r}}}$ $(\underset{\alpha}{\overset{(\mathring{-})}{\mathbf{r}}})$ are the radius-vectors of the surfaces $\underset{\alpha}{\overset{(+)}{S}}$ $(\underset{\alpha}{\overset{(\mathring{+})}{S}})$ and $\underset{\alpha}{\overset{(-)}{S}}$ $(\underset{\alpha}{\overset{(\mathring{-})}{S}})$, respectively, in the deformed (non-deformed) state of the multilayer thin body. It is not difficult to see that in this case (under a slipping of absolutely smooth contacted surfaces) we have the following relations instead of (8):

$$\begin{gathered}\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha}{\overset{(\mathring{+})}{\mathbf{r}}}(x^1, x^2) + \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1, x^2), \ \ \underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha}{\overset{(\mathring{-})}{\mathbf{r}}}(x^1, x^2) + \underset{\alpha}{\overset{(-)}{\mathbf{u}}}(x^1, x^2), \ \ \alpha = \overline{1, K}, \\ \underset{\alpha}{\overset{(\mathring{+})}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha+1}{\overset{(\mathring{-})}{\mathbf{r}}}(x^1, x^2), \ \ \underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) \neq \underset{\alpha+1}{\overset{(-)}{\mathbf{r}}}(x^1, x^2), \ \ \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1, x^2) \neq \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1, x^2), \\ \underset{\alpha}{\mathbf{v}}(x^1, x^2) = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1, x^2) - \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1, x^2), \ \ \alpha = \overline{1, K-1}.\end{gathered} \tag{9}$$

Obviously, $\underset{\alpha}{\mathbf{v}}(x^1,x^2)$ is the vector of the relative displacement of the corresponding points of the contacted surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha}{\overset{(-)}{S}}$ $(\alpha=\overline{1,K-1})$, being an unknown in the considered case. The absence of the friction between the layers allows us to write the additional relations

$$\begin{gathered}\underset{\alpha+1}{\overset{(-)}{u}}_{(n)} = \underset{\alpha}{\overset{(+)}{u}}_{(n)} \quad (\underset{\alpha}{v}_{(n)}=0), \quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}_{(s)}=0, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}_{(s)}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}_{(n)} = -\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}_{(n)},\\ \alpha=\overline{1,K-1},\quad x'\in \underset{\alpha}{\overset{(+)}{S}}{}^0 \subset \underset{\alpha}{\overset{(+)}{S}},\end{gathered} \tag{10}$$

where $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}_{(s)}$ $(\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}_{(s)})$ and $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}_{(n)}$ $(\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}_{(n)})$ are the tangent and normal components of the stress vector (the interaction force intensity) $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}$ $(\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}})$, i.e., $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}=\underset{\alpha}{\overset{(+)}{\mathbf{P}}}_{(s)}+\underset{\alpha}{\overset{(+)}{\mathbf{P}}}_{(n)}$, $\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}=\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}_{(s)}+\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}_{(n)}$,, $\alpha=\overline{1,K-1}$. It is not difficult to see that (10) implies the relations

$$\begin{gathered}\underset{\alpha+1}{\overset{(-)}{u}}_{(n)}=\underset{\alpha}{\overset{(+)}{u}}_{(n)},\ \ \underset{\alpha}{v}_{(n)}=0),\ \ \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\vartheta})\cdot\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}=0,\ \ \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}}=0,\\ \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}}=\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}},\ \ \alpha=\overline{1,K-1},\ \ x'\in\underset{\alpha}{\overset{(+)}{S}}{}^0\subset\underset{\alpha}{\overset{(+)}{S}}.\end{gathered} \tag{11}$$

Here the notation $\underset{\widetilde{\alpha}}{\overset{(\sim)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\vartheta})$, $\sim\,\in\{-,+\}$ means the dependence of $\underset{\widetilde{\alpha}}{\overset{(\sim)}{\mathbf{P}}}$ on $\underset{\alpha}{\mathbf{u}}$ and $\underset{\alpha}{\vartheta}$, and $\underset{\widetilde{\alpha}}{\overset{(-)}{\mathbf{P}}}=\underset{\widetilde{\alpha}}{\mathbf{P}}|_{x^3=0}$, $\underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}=\underset{\widetilde{\alpha}}{\mathbf{P}}|_{x^3=1}$. The corresponding relations are obtained from the first equality of (II: 21) with $\underset{\widetilde{\widetilde{\alpha}}}{\mathbf{A}}=0$, $\underset{\alpha}{\boldsymbol{\varphi}}=0$. In the case considered here, relations (11) close the system of equations of the classic theory of multilayer thin bodies. In the case of the micropolar theory of multilayer thin bodies equalities (11) should be replaced by the following ones:

$$\begin{gathered}\underset{\alpha+1}{\overset{(-)}{u}}_{(n)}=\underset{\alpha}{\overset{(+)}{u}}_{(n)},\quad \underset{\alpha+1}{\overset{(-)}{\varphi}}_{(n)}=\underset{\alpha}{\overset{(+)}{\varphi}}_{(n)},\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\boldsymbol{\varphi}},\underset{\alpha+1}{\vartheta})\cdot\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}=0,\\ \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\boldsymbol{\varphi}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\boldsymbol{\varphi}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}}=\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\boldsymbol{\varphi}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}},\\ \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha+1}}{\overset{(-)}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}})\cdot\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha}}{\overset{(+)}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}}=0,\\ \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha}}{\overset{(+)}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}}=\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\widetilde{\alpha+1}}{\overset{(-)}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}},\quad \alpha=\overline{1,K-1},\quad x'\in\underset{\alpha}{\overset{(+)}{S}}{}^0\subset\underset{\alpha}{\overset{(+)}{S}}.\end{gathered} \tag{12}$$

In this case, along with the vector of relative displacement, the vector of relative rotation $\underset{\alpha}{\boldsymbol{\psi}}=\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}-\underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}}$ of the corresponding points of the contacted surfaces is introduced into consideration; $\underset{\alpha+1}{\overset{(-)}{\varphi}}_{(n)}$ and $\underset{\alpha}{\overset{(+)}{\varphi}}_{(n)}$ are the normal components of the vectors $\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}$ and $\underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}}$, respectively. Note that relations (12) are written subject to the fact that each layer has a center of symmetry, i.e. the tensors $\underset{\widetilde{\widetilde{\alpha}}}{\mathbf{A}}=0$ and $\underset{\widetilde{\widetilde{\alpha}}}{\mathbf{B}}=0$ in (II: 21) for all α. If this is not the case, then these relations should be replaced by other ones depending on the considered governing relations. For example, if relations (II: 21) are considered as governing ones for a material having no center of symmetry, then instead of (12) we have

$$
\begin{gathered}
\underset{\alpha+1}{\overset{(-)}{u}}{}_{(n)} = \underset{\alpha}{\overset{(+)}{u}}{}_{(n)}, \quad \underset{\alpha+1}{\overset{(-)}{\varphi}}{}_{(n)} = \underset{\alpha}{\overset{(+)}{\varphi}}{}_{(n)}, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha+1}{\overset{(-)}{\mathbf{s}}} = 0, \\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}} = 0, \quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}} = \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}}, \\
\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} = 0, \quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}} = 0, \\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}} = \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}}, \quad \alpha = \overline{1, K-1}, \quad x' \in \underset{\alpha}{\overset{(+)}{S}}{}^{0} \subset \underset{\alpha}{\overset{(+)}{S}}.
\end{gathered} \tag{13}
$$

Note also that the contact conditions should be supplied with the conditions of heat content on the contacted surfaces, which is not difficult. Therefore, in order to shorten the presentation, we do not consider them here.

2.4. Conditions under relative displacement of points of uneven contacted surfaces of layers

In the case considered here, the slipping with the friction of layers with respect to each other can take place in the process of deformation of the multilayer thin body. The relative slipping does not occur until the magnitude of the tangent component of the interaction force $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(s)}$ $(\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(s)})$ (force of friction) between the contacted surfaces reaches its limit (maximal possible) value $|\underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}^{*}|$ $(|\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}^{*}|)$, therefore, $\underset{\alpha}{\mathbf{v}}(x^1, x^2) = 0$, $\alpha = \overline{1, K-1}$. When the force of friction reaches its limit value, the slipping begins, and the relations presented above should be replaced by other ones. First of all, note that for the case of the classic theory of multilayer thin bodies instead of (11) we have

$$
\begin{gathered}
\underset{\alpha+1}{\overset{(-)}{u}}{}_{(n)} = \underset{\alpha}{\overset{(+)}{u}}{}_{(n)}, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha+1}{\overset{(-)}{\mathbf{s}}} = \underset{\alpha+1}{\overset{(-)}{P}}{}^{*}_{(s)}, \quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}} = \underset{\alpha}{\overset{(+)}{P}}{}^{*}_{(s)}, \\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}} = \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}}, \quad \alpha = \overline{1, K-1}, \quad x' \in \underset{\alpha}{\overset{(+)}{S}}{}^{0} \subset \underset{\alpha}{\overset{(+)}{S}},
\end{gathered} \tag{14}
$$

and in the case of the micropolar theory of multilayer thin bodies whose layers do not have a center of symmetry, we assume the following relations instead of (13):

$$
\begin{gathered}
\underset{\alpha+1}{\overset{(-)}{u}}{}_{(n)} = \underset{\alpha}{\overset{(+)}{u}}{}_{(n)}, \quad \underset{\alpha+1}{\overset{(-)}{\varphi}}{}_{(n)} = \underset{\alpha}{\overset{(+)}{\varphi}}{}_{(n)}, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha+1}{\overset{(-)}{\mathbf{s}}} = \underset{\alpha+1}{\overset{(-)}{P}}{}^{*}_{(s)}, \\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}} = \underset{\alpha}{\overset{(+)}{P}}{}^{*}_{(s)}, \quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}} = \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}}, \\
\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}} = \underset{\alpha+1}{\overset{(-)}{\mu}}{}^{*}_{(n)}, \quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{n}}} = \underset{\alpha}{\overset{(+)}{\mu}}{}^{*}_{(n)}, \\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha}}{\overset{(+)}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}} = \underset{\alpha}{\overset{(+)}{\mathbf{n}}} \cdot \underset{\widetilde{\alpha+1}}{\overset{(-)}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \underset{\alpha}{\overset{(+)}{\mathbf{s}}}, \quad \alpha = \overline{1, K-1}, \quad x' \in \underset{\alpha}{\overset{(+)}{S}}{}^{0} \subset \underset{\alpha}{\overset{(+)}{S}}.
\end{gathered} \tag{15}
$$

Here, naturally, $\overset{(+)}{\underset{\alpha}{\mu}}{}^{*}_{(n)} = \overset{(+)}{\underset{\alpha}{\boldsymbol{\mu}}}{}^{*} \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}}$, $\overset{(-)}{\underset{\alpha+1}{\mu}}{}^{*}_{(n)} = \overset{(-)}{\underset{\alpha+1}{\boldsymbol{\mu}}}{}^{*} \cdot \overset{(+)}{\underset{\alpha+1}{\mathbf{n}}}$, where $\overset{(+)}{\underset{\alpha}{\boldsymbol{\mu}}}{}^{*}$ $(\overset{(-)}{\underset{\alpha+1}{\boldsymbol{\mu}}}{}^{*})$ is the intensity of the limit momentum. Therefore, $\overset{(+)}{\underset{\alpha}{P}}{}^{*}_{(s)}$, $\overset{(-)}{\underset{\alpha+1}{P}}{}^{*}_{(s)}$, $\overset{(+)}{\underset{\alpha}{\mu}}{}^{*}_{(n)}$ and $\overset{(-)}{\underset{\alpha+1}{\mu}}{}^{*}_{(n)}$ are unknown values in relations (14) and (15) determined from some a priori dependencies, conditions of slipping with friction, which, generally speaking, must depend on geometric and physical-mechanical properties of contacted bodies. In the classic case we may suppose the relations hold

$$\mathbf{L}(x^1, x^2, \mathbf{v}_s, \dot{\mathbf{v}}_s, [T], \mathbf{P}^{(l)*}, \dots) = 0, \tag{16}$$

where $\mathbf{v}_s$ and $\dot{\mathbf{v}}_s$ are the tangent components of the vectors of the relative displacement and relative velocity, $[T]$ is the temperature jump, $\mathbf{P}^{(l)*}$ is the limit stress vector on a plane element with the normal $\mathbf{l}$, the ellipsis denotes the dependence on some other parameters. Based on (16), we can accept that the generalized model of Coulomb friction is valid:

$$\mathbf{P}^*_{(s)} = \underset{\sim}{\mathbf{f}}(x^1, x^2, [T], \mathbf{P}^*_{(n)}) \cdot \dot{\mathbf{v}}_s, \tag{17}$$

which takes into account the anisotropy of the friction. Here $\mathbf{P}^*_{(s)}$ and $\mathbf{P}^*_{(n)}$ are the limit tangent and normal components of the stress vector $\mathbf{P}^{(l)*}$. The second rank tensor $\underset{\sim}{\mathbf{f}}(x^1, x^2, [T], \mathbf{P}^*_{(n)})$ is called the tensor of friction coefficients. Obviously, in the isotropic case we have $\underset{\sim}{\mathbf{f}} = f\underset{\sim}{\mathbf{E}}$, where $\underset{\sim}{\mathbf{E}}$ is the unit second rank tensor. Representing (17) for contacted surfaces of a multi-layer thin body, we obtain the missing required relations. Based on similar arguments in the case of the micropolar theory, we can assert that the following a priori relations are valid:

$$\begin{aligned} &\mathbf{L}(x^1, x^2, \mathbf{v}_s, \dot{\mathbf{v}}_s, \boldsymbol{\psi}_n, \dot{\boldsymbol{\psi}}_n, [T], \mathbf{P}^{(l)*}, \dots) = 0, \\ &\mathbf{M}(x^1, x^2, \mathbf{v}_s, \dot{\mathbf{v}}_s, \boldsymbol{\psi}_n, \dot{\boldsymbol{\psi}}_n, [T], \boldsymbol{\mu}^{(l)*}, \dots) = 0, \end{aligned} \tag{18}$$

where $\boldsymbol{\psi}_n$ and $\dot{\boldsymbol{\psi}}_n$ are the normal components of the vectors of the relative internal rotation and relative internal rotation velocity of adjacent layers, $\boldsymbol{\mu}^{(l)*}$ is the limit vector of the couple stress on a plane element with the normal $\mathbf{l}$, the other parameters are the same as in (16). Based on (18) and similar to (17), for the micropolar theory we can assume that the following relations are valid:

$$\begin{aligned} &\mathbf{P}^*_{(s)} = \underset{\sim}{\mathbf{f}}(x^1, x^2, [T], \mathbf{P}^*_{(n)}) \cdot \dot{\mathbf{v}}_s + \underset{\sim}{\mathbf{h}}(x^1, x^2, [T], \mathbf{P}^*_{(n)}) \cdot \dot{\boldsymbol{\psi}}_n, \\ &\boldsymbol{\mu}^*_{(n)} = \underset{\sim}{\mathbf{g}}(x^1, x^2, [T], \boldsymbol{\mu}^*_{(s)}) \cdot \dot{\boldsymbol{\psi}}_n + \underset{\sim}{\mathbf{l}}(x^1, x^2, [T], \boldsymbol{\mu}^*_{(s)}) \cdot \dot{\mathbf{v}}_s, \end{aligned} \tag{19}$$

that take into account the anisotropy of the friction. Here $\underset{\sim}{\mathbf{f}}$, $\underset{\sim}{\mathbf{h}}$, $\underset{\sim}{\mathbf{g}}$ and $\underset{\sim}{\mathbf{l}}$ are the second rank tensors called the tensors of friction coefficients. Therefore, in the case of an isotropic friction we have $\underset{\sim}{\mathbf{f}} = f\underset{\sim}{\mathbf{E}}$, $\underset{\sim}{\mathbf{h}} = h\underset{\sim}{\mathbf{E}}$, $\underset{\sim}{\mathbf{g}} = g\underset{\sim}{\mathbf{E}}$ and $\underset{\sim}{\mathbf{l}} = l\underset{\sim}{\mathbf{E}}$, where $\underset{\sim}{\mathbf{E}}$ is the unit second rank tensor. It should be noted here that the coefficients of friction are determined by experiments and are given in tables. The author knows little in this direction for the micropolar theory, but for the classic theory these coefficients can be obtained, e.g., from [12, 13, 14]. Representing (19) for the contacted surfaces of a multilayer thin body, we get the missing required relations in the case of the micropolar theory.

2.5. Conditions under a partial exfoliation of contacted surfaces of layers

For the classic theory of multilayer thin bodies in this case we have the conditions

$$\begin{gathered}\underset{\alpha}{\mathbf{v}}(x^1,x^2) = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1,x^2) - \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2) \neq 0, \quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}(x^1,x^2) = 0, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}(x^1,x^2) = 0,\\ (x^1,x^2) \subset \underset{\alpha}{\overset{(+)}{S}}{}^0 \subset \underset{\alpha}{\overset{(+)}{S}}, \quad \alpha = \overline{1, K-1},\end{gathered} \tag{20}$$

and for the micropolar theory of multilayer thin bodies we get the conditions

$$\begin{gathered}\underset{\alpha}{\mathbf{v}}(x^1,x^2) = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1,x^2) - \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2) \neq 0, \quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}(x^1,x^2) = 0, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}(x^1,x^2) = 0,\\ \underset{\alpha}{\boldsymbol{\psi}}(x^1,x^2) = \underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}(x^1,x^2) - \underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}}(x^1,x^2) \neq 0, \quad \underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}}(x^1,x^2) = 0, \quad \underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}(x^1,x^2) = 0,\\ (x^1,x^2) \subset \underset{\alpha}{\overset{(+)}{S}}{}^0 \subset \underset{\alpha}{\overset{(+)}{S}}, \quad \alpha = \overline{1, K-1}.\end{gathered} \tag{21}$$

Note that if $\underset{\alpha}{\overset{(+)}{S}}{}^0 = \underset{\alpha}{\overset{(+)}{S}}$, then we have a complete exfoliation of contacted layers. Other conditions posed on deformed and force states of exterior surfaces of multilayer thin bodies are also possible: a contact with rigid or elastic bodies, a forced displacement of points, etc. Note also that based on [5, 15] and quite similarly to this paper, one can construct micropolar theories of multilayer thin bodies with two small sizes and those of plane domains with one small size, respectively (it remains to write down the corresponding relations). We do not pay attention to this in the present paper.

In conclusion we note that the other important direction is the study of eigenvalue problems for the tensor and tensor-block matrix of any even rank, since the constitutive relations for most classical and micropolar media of different rheology (here, of course, also include porous and multilayer textile media) are written using a tensor and tensor-block matrix of even rank, respectively. Several questions concerning these problems, as well as tensor calculus, have been studied in some detail in [16, 19, 18, 20, 17]. A very important direction is also the investigation of internal structures of differential tensors-operators and tensor-block matrix operators of even rank. This is due to the fact that such operators are operators of systems of equations of motion and static boundary conditions with respect to kinematic characteristics (displacement, rotation, velocity, angular velocity) for most classical and micropolar mediums. The study of these problems promotes the decomposition of initial-boundary value problems in the case of linear theories. Some questions about the decomposition of initial-boundary value problems can be found in [1, 2, 21].

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