Formulation of the initial boundery velue problems in the theory of multilayer thermoelastic thin bodies in moments (part I)

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Abstract: In the paper the new parametrization of a multilayer thin domain is applied. In contrast with classic approaches, several base surfaces and an analytic method with application of orthogonal polynomial systems are used. Geometric characteristics typical for the proposed parametrizations are considered. The new parametrization in the case of a one-layered thin body is described in detail in [5–10]. Various representations of the equations of motion, the heat influx, the constitutive relations of physical and heat content are given for the new body domain parametrization. The definition of the kth order moment of a certain quantity with respect to an orthonormal polynomial systems is given. The expressions of moments of first- and second-order partial derivatives of a certain tensor field are obtained and this is also done for some important expressions required for constructing different variants of the theory of thin body. Various variants of the equations of motion in moments with respect to orthogonal polynomial systems are also obtained. The interlayer conditions are written down under various connections of adjacent layers of a multilayer body. Formulation of the initial boundery velue problems in the theory of multilayer thermoelastic thin bodies in moments are given.

1. Parametrization of a multilayer thin domain of the three-dimensional Euclidean space with several base surfaces

Consider a multilayer thin domain of the Euclidean space consisting of not more than countably many layers. We perform the parametrization of this domain in the same way as in [1,2]. Let the layers be enumerated in the ascending order, i.e., for example, if $\alpha \geq 2$ is the serial number of a certain layer, then the serial number of the previous layer is $\alpha - 1$ and the serial number of the next layer is $\alpha + 1$. Each layer has two frontal surfaces. The frontal surface of the layer α, which lies to the side of the previous layer $\alpha - 1$, is called the interior base surface and denoted by $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$, whereas the frontal surface of the layer α, which lies to the side of the next layer $\alpha + 1$, is called the exterior base surface and denoted by $\underset{\alpha}{\overset{(+)}{\mathrm{S}}}$.

If the multilayer structure consists of K layers, then $\underset{1}{\overset{(-)}{\mathrm{S}}}$ ($\underset{1}{\overset{(+)}{\mathrm{S}}}$) and $\underset{K}{\overset{(-)}{\mathrm{S}}}$ ($\underset{K}{\overset{(+)}{\mathrm{S}}}$) are the interior and exterior surfaces of the first and last layers, respectively. In this case, $\underset{1}{\overset{(-)}{\mathrm{S}}}$ and $\underset{K}{\overset{(+)}{\mathrm{S}}}$ are also called the interior and exterior surfaces of the multilayer structure.

We assume that the frontal surfaces of each layer are regular surfaces and its lateral surface is a ruled surface in the case where the layer is bounded and unclosed.

Note that the analytic method with the use of the Legendre polynomial system in constructing the one-layer thin body theory [11–39] and multilayer thin body theory [40–49] was also applied by other authors. In this direction the author had published the papers [1–8, 50–59] and others with the application of Legendre and Chebyshev polynomial systems. These expansions can be successfully used in constructing any thin body theory. Despite this, the classic theories constructed by this method are far to be complete, and the more so, the micropolar theories and theories of other rheology are.

1.1. Vector parametric equation of the layer α and the system of vector parametric equations of a multilayer thin domain

The radius-vector of an arbitrary point $\underset{\alpha}{\mathrm{M}}$ of the layer α has the form

$$\underset{\alpha}{\mathbf{r}}(x^1,x^2,x^3)=\underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)+x^3\underset{\alpha}{\mathbf{h}}(x^1,x^2)=(1-x^3)\underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)+x^3\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2) \tag{1}$$

for all $\alpha\in\mathbb{N}$ and $\forall\, x^3\in[0,1]$, where the vector relations

$$\underset{\alpha}{\overset{(-)}{\mathbf{r}}}=\underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2),\quad \underset{\alpha}{\overset{(+)}{\mathbf{r}}}=\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2),\quad \alpha\in\mathbb{N}, \tag{2}$$

are the vector equations of the base surfaces $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$ and $\underset{\alpha}{\overset{(+)}{\mathrm{S}}}$, respectively, x^1, x^2 are curvilinear (or Gaussian) coordinates on the interior base surface $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$, and $\mathbb{N}$ is the set of natural numbers. The vector $\underset{\alpha}{\mathbf{h}}(x^1,x^2)=\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)-\underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)$, which topologically maps the interior base surface $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$ onto the exterior base surface $\underset{\alpha}{\overset{(+)}{\mathrm{S}}}$, in general, is not orthogonal to the base surfaces, and, moreover, the endpoint of each $\underset{\alpha}{\mathbf{h}}(x^1,x^2)$ is the initial point of $\underset{\alpha+1}{\mathbf{h}}(x^1,x^2)$, $\forall\alpha$, i.e., the following relation holds:

$$\begin{aligned}\underset{\alpha+\delta}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)&=\underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha}^{\alpha+\delta}\underset{\nu}{\mathbf{h}}=\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha+1}^{\alpha+\delta}\underset{\nu}{\mathbf{h}}=\underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)\\&+\sum_{\nu=\alpha}^{\alpha+\delta}\Big[\underset{\nu}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)-\underset{\nu}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)\Big]=\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha+1}^{\alpha+\delta}\Big[\underset{\nu}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)-\underset{\nu}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)\Big],\ \ \forall\alpha,\delta.\end{aligned} \tag{3}$$

Let a multilayer domain[1] consist of K layers. Then introducing the notation

[1] We use the usual rules of tensor calculus [9, 10, 60–63]. We mainly preserve the notation and conventions of the previous works. Under symbols, we write indices denoting the serial numbers of layers . The Greek indices under symbols assume their values according to circumstances, and capital and small Latin indices assume the values 1, 2 and 1, 2, 3, respectively. A record of the form (I: 24) means a reference to formula (24) from the first part of the paper, and (II: 26) means a reference to formula (26) from the second part of the work.

$$\mathbf{h} = \sum_{\nu=1}^{K} \underset{\nu}{\mathbf{h}} = \sum_{\nu=1}^{K} \left[\underset{\nu}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) - \underset{\nu}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) \right], \tag{4}$$

we have

$$\underset{K}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) = \underset{1}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) + \mathbf{h}(x^1, x^2) = \underset{1}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) + \sum_{\nu=1}^{K} \left[\underset{\nu}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) - \underset{\nu}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) \right]. \tag{5}$$

Note that Eq. (1) is the vector parametric equation of the layer α for a fixed α, and when α varies in the corresponding range and conditions (3) hold, the system of vector parametric equations of the multilayer thin domain considered. It is easy to see that (1) for any x^1, x^2 and $x^3 = 0$ defines the interior base surface $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$, and for any x^1, x^2 and $x^3 = 1$, it defines the exterior lateral surface $\underset{\alpha}{\overset{(+)}{\mathrm{S}}}$, whereas for any x^1, x^2 and $x^3 = \mathrm{const}$,where $x^3 \in (0, 1)$, it defines the equidistance surface for the base surfaces $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$ and $\underset{\alpha}{\overset{(+)}{\mathrm{S}}}$, which is denoted by $\underset{\alpha}{\mathrm{S}}$.

1.2. Two-dimensional families of bases and the families of parametrizations of the surface of the layer α generated by them

For the derivatives of relations (1) and (2) in x^P at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$ $\forall\alpha$, let us introduce the notation

$$\underset{\alpha}{\mathbf{r}}_P \equiv \partial_P \underset{\alpha}{\mathbf{r}} \equiv \partial_P \underset{\alpha}{\mathbf{r}} / \partial x^P, \quad \underset{\alpha P}{\mathbf{r}_{\star}} \equiv \partial_P \underset{\alpha}{\overset{(\star)}{\mathbf{r}}} \equiv \partial \underset{\alpha}{\overset{(\star)}{\mathbf{r}}} / \partial x^P, \quad \star \in \{-, +\},\ \forall\alpha. \tag{6}$$

The pair of vectors $\underset{\alpha\,1}{\mathbf{r}_{\star}}\ \underset{\alpha\,2}{\mathbf{r}_{\star}}$, $\star \in \{-, \emptyset, +\}$ $\forall\alpha$ defined at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{S}}$, $\star \in \{-, \emptyset, +\}$, $\forall\alpha$ obviously compose two-dimensional covariant surface bases, and $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}}\,\underset{\alpha\,1}{\mathbf{r}_{\star}}\ \underset{\alpha\,2}{\mathbf{r}_{\star}}$, $\star \in \{-, \emptyset, +\}$ $\forall\alpha$ are two-dimensional covariant surface frames, which, in turn, generate the corresponding parametrizations of the surfaces considered. As is known [60–63] (see also [9, 10]), according to these frames (bases), we can construct the corresponding contravariant frames $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}}\ \underset{\alpha}{\mathbf{r}}^{\overset{\star}{1}}\ \underset{\alpha}{\mathbf{r}}^{\overset{\star}{2}}$ (bases $\underset{\alpha}{\mathbf{r}}^{\overset{\star}{1}}\ \underset{\alpha}{\mathbf{r}}^{\overset{\star}{2}}$), $\star \in \{-, \emptyset, +\}$, $\forall\alpha$. Naturally, the covariant and contravariant bases generate the geometric characteristics inherit for them. Defining the frames (bases) at each point of the surfaces $\underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$, $\forall\alpha$, we obtain the corresponding families of frames (bases), which, in turn, generate the corresponding parametrizations.

1.3. Three-dimensional families of bases and the families of parametrization of the domain of the layer α generated by them

Taking into account the expression of the radius-vector $\underset{\alpha}{\mathbf{r}}$ (1) in the first relation in (6) and introducing the notation $\underset{\alpha}{\mathbf{h}}_P \equiv \partial \mathbf{h} / \partial x^P \equiv \partial_P \underset{\alpha}{\mathbf{h}}$, we obtain

$$\underset{\alpha}{\mathbf{r}}_P = \underset{\alpha P}{\mathbf{r}_{-}} + x^3 \underset{\alpha}{\mathbf{h}}_P = (1 - x^3) \underset{\alpha P}{\mathbf{r}_{-}} + x^3 \underset{\alpha P}{\mathbf{r}_{+}},\ \forall\,\alpha. \tag{7}$$

Now, differentiating (1) in x^3, we have

$$\underset{\alpha}{\mathbf{r}}_3 \equiv \partial_3 \underset{\alpha}{\mathbf{r}} \equiv \partial \underset{\alpha}{\mathbf{r}}/\partial x^3 = \underset{\alpha}{\mathbf{h}}(x^1, x^2), \quad \forall x^3 \in [0,1], \quad \forall \alpha. \tag{8}$$

According to (8), we assume that

$$\underset{\alpha}{\mathbf{r}}_{\overset{-}{3}} \equiv \underset{\alpha}{\mathbf{r}}_3 \equiv \underset{\alpha}{\mathbf{r}}_{\overset{+}{3}} \equiv \partial_3 \underset{\alpha}{\mathbf{r}} = \underset{\alpha}{\mathbf{h}}(x^1, x^2), \quad \forall x^3 \in [0,1], \quad \forall \alpha. \tag{9}$$

Relation (9) allows us to define the spatial covariant bases $\underset{\alpha}{\mathbf{r}}_{\overset{\star}{p}}$, $\star \in \{-, +\}$, $\forall\, \alpha$ at the points $\underset{\alpha}{\overset{(\star)}{M}} \in \underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, +\}$, $\forall\, \alpha$, respectively. Therefore, the third basis vector of the spatial covariant bases at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$, for each layer α is the same vector $\underset{\alpha}{\mathbf{h}}(x^1, x^2)$. In view of (9), we can join relations (7)and (8) and represent them as

$$\underset{\alpha}{\mathbf{r}}_p = \underset{\alpha}{\mathbf{r}}_{\overset{-}{p}} + x^3 \underset{\alpha}{\mathbf{h}}_p = (1 - x^3)\underset{\alpha}{\overset{(-)}{\mathbf{r}}}_{\overset{-}{p}} + x^3 \underset{\alpha}{\overset{(+)}{\mathbf{r}}}_{\overset{+}{p}}, \quad \forall \alpha. \tag{10}$$

The triples of vectors $\underset{\alpha}{\mathbf{r}}_{\overset{\star}{1}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{2}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{3}}$, $\star \in \{-, \emptyset, +\}$, $\forall\, \alpha$ defined at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{S}}$, $\star \in \{-, \emptyset, +\}$, $\forall\, \alpha$ obviously compose three-dimensional covariant spatial bases, and $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{1}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{2}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{3}}$, $\star \in \{-, \emptyset, +\}$, $\forall\, \alpha$ compose three-dimensional spatial covariant frames, which, in turn, generate the corresponding parametrizations. As is known [9, 10, 60–62], according to these frames(bases), we can construct the corresponding contravariant frames $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}}\underset{\alpha}{\mathbf{r}}^{\overset{\star}{1}}\underset{\alpha}{\mathbf{r}}^{\overset{\star}{2}}\underset{\alpha}{\mathbf{r}}^{\overset{\star}{3}}$ (bases $\underset{\alpha}{\mathbf{r}}^{\overset{\star}{1}}\underset{\alpha}{\mathbf{r}}^{\overset{\star}{2}}\underset{\alpha}{\mathbf{r}}^{\overset{\star}{3}}$), $\star \in \{-, \emptyset, +\}$, $\forall\, \alpha$. Indeed, by their definition [9, 10, 60–62], we have

$$\underset{\alpha}{\mathbf{r}}^{\tilde{k}} = \frac{1}{2} \underset{\alpha}{\overset{(\sim)}{\mathrm{C}}}{}^{\tilde{k}\tilde{p}\tilde{q}} \underset{\alpha}{\mathbf{r}}_{\tilde{p}} \times \underset{\alpha}{\mathbf{r}}_{\tilde{q}}, \quad \sim \in \{-, \emptyset, +\},\ \forall\, \alpha, \tag{11}$$

where $\underset{\alpha}{\overset{(\sim)}{\mathrm{C}}}{}^{\tilde{k}\tilde{p}\tilde{q}} = (\underset{\alpha}{\mathbf{r}}^{\tilde{k}} \times \underset{\alpha}{\mathbf{r}}^{\tilde{p}}) \cdot \underset{\alpha}{\mathbf{r}}^{\tilde{q}}$, $\sim \in \{-, \emptyset, +\}$, $\forall\, \alpha$, are contravariant components of the discriminant tensors [60] of the layer α at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$, $\forall\, \alpha$. It is easy to see that (10) is shortly represented in the form

$$\underset{\alpha}{\mathbf{r}}_p = \underset{\alpha}{g}{}_p^{\overset{\star}{q}} \underset{\alpha}{\mathbf{r}}_{\overset{\star}{q}} = \underset{\alpha}{g}{}_{p\overset{\star}{q}} \underset{\alpha}{\mathbf{r}}^{\overset{\star}{q}}, \quad \star \in \{-, +\},\ \forall\, \alpha, \tag{12}$$

where we have introduced the notation

$$\underset{\alpha}{g}{}_{\breve{p}\tilde{q}} = \underset{\alpha}{\mathbf{r}}_{\breve{p}} \cdot \underset{\alpha}{\mathbf{r}}_{\tilde{q}}, \quad \underset{\alpha}{g}{}_{\breve{p}}^{\tilde{q}} = \underset{\alpha}{\mathbf{r}}_{\breve{p}} \cdot \underset{\alpha}{\mathbf{r}}^{\tilde{q}}, \quad \smile \in \{-, \emptyset, +\}, \quad \sim \in \{-, +\}, \quad \forall \alpha. \tag{13}$$

in view of (10) and (13), for $\underset{\alpha}{g}{}_{p\tilde{q}}$ and $\underset{\alpha}{g}{}_p^{\tilde{q}}$, we have

$$\underset{\alpha}{g}{}_{p\check{q}} = (1 - x^3)\underset{\alpha}{g}{}_{\overset{-}{p}\check{q}} + x^3 \underset{\alpha}{g}{}_{\overset{+}{p}\check{q}}, \quad \underset{\alpha}{g}{}_p^{\check{q}} = (1 - x^3)\underset{\alpha}{g}{}^{\check{q}}_{\overset{-}{p}} + x^3 \underset{\alpha}{g}{}^{\check{q}}_{\overset{+}{p}}, \quad \smile \in \{-, +\}, \quad \forall \alpha. \tag{14}$$

Also, it is easy to obtain the expressions for $\underset{\alpha}{g}_{pq}$. Indeed, by (12) and (14), we have

$$\underset{\alpha}{g}_{pq} = \underset{\alpha}{\mathbf{r}}_p \cdot \underset{\alpha}{\mathbf{r}}_q = (1-x^3)^2 \underset{\alpha}{g}_{\overset{-}{p}\overset{-}{q}} + x^3(1-x^3)(\underset{\alpha}{g}_{\overset{-}{p}\overset{+}{q}} + \underset{\alpha}{g}_{\overset{+}{p}\overset{-}{q}}) + (x^3)^2 \underset{\alpha}{g}_{\overset{+}{p}\overset{+}{q}}, \quad \star \in \{-,+\}, \quad \forall\alpha. \tag{15}$$

Let us find the expressions for $\sqrt{\underset{\alpha}{g}} = (\underset{\alpha}{\mathbf{r}}_1 \times \underset{\alpha}{\mathbf{r}}_2) \cdot \underset{\alpha}{\mathbf{r}}_3$. By the first relation in (12), we obtain

$$\sqrt{\underset{\alpha}{g}} = \frac{1}{2}\epsilon^{IJ}(\underset{\alpha}{\mathbf{r}}_I \times \underset{\alpha}{\mathbf{r}}_J) \cdot \underset{\alpha}{\mathbf{r}}_3 = \sqrt{\underset{\alpha}{\overset{(\sim)}{g}}}\det(\underset{\alpha}{g}_{p}^{\tilde{q}}) = \sqrt{\underset{\alpha}{\overset{(\sim)}{g}}}\det(\underset{\alpha}{g}_{P}^{\tilde{Q}}), \quad \sim \in \{-,+\}, \quad \forall\alpha, \tag{16}$$

where ϵ^{IJ}, ϵ_{KL} are the Levi-Civita symbols and

$$\sqrt{\underset{\alpha}{\overset{(\sim)}{g}}} = (\underset{\alpha}{\mathbf{r}}_{\tilde{1}} \times \underset{\alpha}{\mathbf{r}}_{\tilde{2}}) \cdot \underset{\alpha}{\mathbf{r}}_{\tilde{3}}, \quad \sim \in \{-,+\}, \quad \sqrt{\underset{\alpha}{\overset{(-)}{g}}} = \sqrt{\underset{\alpha}{g}}\Big|_{x^3=0}, \quad \sqrt{\underset{\alpha}{\overset{(+)}{g}}} = \sqrt{\underset{\alpha}{g}}\Big|_{x^3=1}, \quad \forall\alpha.$$

In turn, from (16), we have

$$\underset{\alpha}{\overset{(\sim)}{\vartheta}} \equiv \sqrt{\underset{\alpha}{g}\,\underset{\alpha}{\overset{(\sim)}{g}}{}^{-1}} = \frac{1}{2}\,\epsilon^{IJ}\epsilon_{KL}\underset{\alpha}{g}_{I}^{\tilde{K}}\,\underset{\alpha}{g}_{J}^{\tilde{L}} = \det(\underset{\alpha}{g}_{P}^{\tilde{Q}}), \quad \sim \in \{-,+\}, \quad \forall\alpha. \tag{17}$$

Note that analogously to (16), in a more general case, we have

$$\sqrt{\underset{\alpha}{\overset{(\sim)}{g}}} = \frac{1}{2}\sqrt{\underset{\alpha}{\overset{(\smile)}{g}}}\epsilon^{IJ}\epsilon_{KL}\underset{\alpha}{g}_{\tilde{I}}^{\breve{K}}\,\underset{\alpha}{g}_{\tilde{J}}^{\breve{L}} = \sqrt{\underset{\alpha}{\overset{(\smile)}{g}}}\det(\underset{\alpha}{g}_{\tilde{P}}^{\breve{Q}}), \quad \sim, \smile \in \{-,\emptyset,+\}, \quad \forall\alpha. \tag{18}$$

It is easy to see from this that

$$\underset{\alpha}{\overset{(\substack{\smile\\ \sim})}{\vartheta}} \equiv \sqrt{\underset{\alpha}{\overset{(\sim)}{g}}\,\underset{\alpha}{\overset{(\smile)}{g}}{}^{-1}} = \frac{1}{2}\epsilon^{IJ}\epsilon_{KL}\underset{\alpha}{g}_{\tilde{I}}^{\breve{K}}\,\underset{\alpha}{g}_{\tilde{J}}^{\breve{L}} = \det(\underset{\alpha}{g}_{\tilde{P}}^{\breve{Q}}) = \det(\underset{\alpha}{g}_{\tilde{p}}^{\breve{q}}), \quad \smile, \sim \in \{-,\emptyset,+\}, \quad \forall\alpha. \tag{19}$$

It is seen that for $\sim\, = \emptyset$, $\smile \in \{-,+\}$, from (18) we obtain (16), and from (19), we obtain (17). It is easy to verify that by (19), we have

$$\underset{\alpha}{\overset{(\substack{\smile\\ \sim})}{\vartheta}} = \underset{\alpha}{\overset{(\substack{\sim\\ \smile})}{\vartheta}}{}^{-1}, \quad \smile, \sim \in \{-,+\}, \quad \forall\alpha; \qquad \underset{\alpha}{\overset{(\approx)}{\vartheta}} = 1, \quad \sim \in \{-,+\}, \quad \forall\alpha. \tag{20}$$

Using (19), we can write relations (17) in the following more detailed form:

$$\begin{aligned} \underset{\alpha}{\overset{(-)}{\vartheta}} &= \sqrt{\underset{\alpha}{g}\,\underset{\alpha}{\overset{(-)}{g}}{}^{-1}} = (1-x^3)^2 \underset{\alpha}{\overset{(=)}{\vartheta}} + x^3(1-x^3)\underset{\alpha}{g}_{\overset{+}{I}}^{\overset{-}{I}} + (x^3)^2 \underset{\alpha}{\overset{(\mp)}{\vartheta}}, \\ \underset{\alpha}{\overset{(+)}{\vartheta}} &= \sqrt{\underset{\alpha}{g}\,\underset{\alpha}{\overset{(+)}{g}}{}^{-1}} = (1-x^3)^2 \underset{\alpha}{\overset{(\pm)}{\vartheta}} + x^3(1-x^3)\underset{\alpha}{g}_{\overset{-}{I}}^{\overset{+}{I}} + (x^3)^2 \underset{\alpha}{\overset{(\substack{+\\+})}{\vartheta}}, \quad \forall\alpha. \end{aligned} \tag{21}$$

It is easy to express $\underset{\alpha}{\mathbf{r}}^k$, $\forall\alpha$, through the vectors $\underset{\alpha}{\mathbf{r}}_{\tilde{m}}$ or $\underset{\alpha}{\mathbf{r}}^{\tilde{m}}$, $\sim \in \{-,+\}$, $\forall\,\alpha$. Indeed, taking into account the first relation of (12) from (11) for $\sim\, = \emptyset$, we obtain

$$\underset{\alpha}{\mathbf{r}}^{k} = \frac{1}{2}\underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\epsilon^{kpq}\epsilon_{lmn}\underset{\alpha}{g}_{p}^{\tilde{m}}\,\underset{\alpha}{g}_{q}^{\tilde{n}}\,\underset{\alpha}{\mathbf{r}}^{\tilde{l}}, \quad \sim \in \{-,+\}, \quad \forall\,\alpha, \tag{22}$$

where ϵ^{kpq}, ϵ_{lmn} are the Levi-Civita symbols. By (22),we introduce the notation

$$\underset{\alpha}{g}{}^{k}_{\tilde{l}} = \frac{1}{2}\underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\epsilon^{kpq}\epsilon_{lmn}\underset{\alpha}{g}{}^{\tilde{m}}_{p}\underset{\alpha}{g}{}^{\tilde{n}}_{q}, \quad \underset{\alpha}{g}{}^{k\tilde{l}} = \frac{1}{2}\underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\epsilon^{kpq}\epsilon_{smn}\underset{\alpha}{g}{}^{\tilde{m}}_{p}\underset{\alpha}{g}{}^{\tilde{n}}_{q}\underset{\alpha}{g}{}^{\tilde{s}\tilde{l}}, \quad \sim\in\{-,+\}, \quad \forall\alpha. \tag{23}$$

Using this notation, we represent relation (22) in the desired form

$$\underset{\alpha}{\mathbf{r}}{}^{p} = \underset{\alpha}{g}{}^{p}_{\tilde{q}}\underset{\alpha}{\mathbf{r}}{}^{\tilde{q}} = \underset{\alpha}{g}{}^{p\tilde{q}}\underset{\alpha}{\mathbf{r}}{}_{\tilde{q}}, \quad \sim\in\{-,+\}, \quad \forall\alpha. \tag{24}$$

It is easy to see that from the first relation in (23), we have

$$\underset{\alpha}{g}{}^{K}_{\tilde{K}} = \underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\underset{\alpha}{g}{}^{\tilde{I}}_{I}, \quad \Rightarrow \quad \underset{\alpha}{g}{}^{\breve{K}}_{\tilde{K}} = \underset{\alpha}{\overset{(\tilde{\smile})}{\vartheta}}{}^{-1}\underset{\alpha}{g}{}^{\tilde{I}}_{\breve{I}} = \underset{\alpha}{\overset{(\underset{\sim}{\smile})}{\vartheta}}\underset{\alpha}{g}{}^{\tilde{I}}_{\breve{I}}, \quad \sim\in\{-,+\}, \quad \forall\,\alpha. \tag{25}$$

Note that in writing the second relation in (25), (19) and (20) were taken into account. Also, let us consider the following objects (matrices):

$$\underset{\alpha\beta}{g}{}_{\breve{p}\,\cdot}^{\,\cdot\,\tilde{q}} = \underset{\alpha}{\mathbf{r}}{}_{\breve{p}}\cdot\underset{\beta}{\mathbf{r}}{}^{\tilde{q}}, \quad \smile,\sim\in\{-,\emptyset,+\}, \quad \forall\alpha,\beta, \tag{26}$$

and the objects obtained from (26) by alternating the indices. We calculate that the number of such objects is equal to 36. It is easy to see that for $\alpha=\beta$ (26) contains (13), (15) and (23). Indeed, from (26), we have

$$\underset{\alpha}{g}{}^{\tilde{q}}_{\breve{p}} = \underset{\alpha\alpha}{g}{}_{\breve{p}\,\cdot}^{\,\cdot\,\tilde{q}} = \underset{\alpha}{\mathbf{r}}{}_{\breve{p}}\cdot\underset{\alpha}{\mathbf{r}}{}^{\tilde{q}}, \quad \smile,\sim\in\{-,\emptyset,+\}, \quad \forall\alpha, \tag{27}$$

and alternating the indices, we obviously obtain the objects considered above and also $\underset{\alpha}{g}{}^{pq} = \underset{\alpha}{\mathbf{r}}{}^{p}\cdot\underset{\alpha}{\mathbf{r}}{}^{q}$, $\forall\alpha$, i.e., in this case, the number of the introduced quantities is equal to 36. It is easy to see that by (26) and (27), the connections between the families of bases are represented in the form

$$\underset{\alpha}{\mathbf{r}}{}_{\tilde{p}} = \underset{\alpha}{g}{}^{\breve{n}}_{\tilde{p}}\underset{\alpha}{\mathbf{r}}{}_{\breve{n}} = \underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}\underset{\beta}{\mathbf{r}}{}_{\breve{n}}, \quad \smile,\sim\in\{-,\emptyset,+\}, \quad \forall\alpha,\beta, \tag{28}$$

which remains valid under index alternation. By (28), it we show that the relation holds

$$\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{q}} = \underset{\alpha\delta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\overset{\star}{n}}\underset{\beta\beta}{g}{}_{\overset{\star}{n}\,\cdot}^{\,\cdot\,\breve{q}}, \quad \smile,\sim,\star\in\{-,\emptyset,+\}, \quad \forall\alpha,\beta,\delta. \tag{29}$$

Differentiating (3)–(5) in x^I and taking into account (28), we obtain

$$\begin{aligned}
&\underset{\alpha+\beta}{\mathbf{r}}{}_{\overset{+}{I}} = \underset{\alpha}{\mathbf{r}}{}_{\overset{-}{I}} + \sum_{\nu=\alpha}^{\alpha+\beta}\Big[\underset{\nu}{g}{}^{\bar{k}}_{\overset{+}{I}} - \underset{\nu}{g}{}^{\bar{k}}_{\overset{-}{I}}\Big]\underset{\nu}{\mathbf{r}}{}_{\overset{-}{k}} = \underset{\alpha}{\mathbf{r}}{}_{\overset{+}{I}} + \sum_{\nu=\alpha+1}^{\alpha+\beta}\Big[\underset{\nu}{g}{}^{\bar{k}}_{\overset{+}{I}} - \underset{\nu}{g}{}^{\bar{k}}_{\overset{-}{I}}\Big]\underset{\nu}{\mathbf{r}}{}_{\overset{-}{k}},\\
&\partial_I\mathbf{h}(x^1,x^2) = \sum_{\nu=1}^{N}\partial_I\underset{\nu}{\mathbf{h}}(x^1,x^2) = \sum_{\nu=1}^{N}\Big[\underset{\nu}{g}{}^{\bar{k}}_{\overset{+}{I}}(x^1,x^2) - \underset{\nu}{g}{}^{\bar{k}}_{\overset{-}{I}}(x^1,x^2)\Big]\underset{\nu}{\mathbf{r}}{}_{\overset{-}{k}}(x^1,x^2),\\
&\underset{N}{\mathbf{r}}{}_{\overset{+}{I}}(x^1,x^2) = \underset{1}{\mathbf{r}}{}_{\overset{-}{I}}(x^1,x^2) + \sum_{\nu=1}^{N}\Big[\underset{\nu}{g}{}^{\bar{k}}_{\overset{+}{I}}(x^1,x^2) - \underset{\nu}{g}{}^{\bar{k}}_{\overset{-}{I}}(x^1,x^2)\Big]\underset{\nu}{\mathbf{r}}{}_{\overset{-}{k}}(x^1,x^2).
\end{aligned} \tag{30}$$

Naturally, all spatial covariant and contravariant bases constructed above have geometric characteristics specific for the parametrizations generated by them. Defining the spatial frames (bases) at each point of the surfaces $\underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$, $\forall\,\alpha$, we obtain the corresponding families of spatial frames (bases), which, in turn generate the corresponding families of parametrizations. Once more, we note that the structure of $\underset{\alpha}{\overset{(\sim)}{\mathrm{S}}}\,{}_{\underset{\alpha}{\overset{(\sim)}{g}}}$-families frames (bases), $\sim\, \in \{-, \emptyset, +\}$, $\forall\,\alpha$, is such that the third basis vectors $\underset{\alpha}{\mathbf{r}}_{\tilde{3}} = \underset{\alpha}{\mathbf{h}}(x^1, x^2)$, $\sim\, \in \{-, \emptyset, +\}$, $\forall\,\alpha$ are not perpendicular to the corresponding base surface $\underset{\alpha}{\overset{(\sim)}{\mathrm{S}}}$, $\sim\, \in \{-, \emptyset, +\}$, $\forall\,\alpha$ in general. However, in a particular case, they can be perpendicular, and in a more particular case, they can be unit normal vectors to the surfaces $\underset{\alpha}{\overset{(\sim)}{\mathrm{S}}}$, $\sim\, \in \{-, \emptyset, +\}$, $\forall\,\alpha$, which are denoted by $\underset{\alpha}{\overset{(\sim)}{n}}$, $\sim\, \in \{-, \emptyset, +\}$, $\forall\,\alpha$, respectively.

The corresponding relations for a one-layered thin body under a new parametrization, as well as for other parametrizations considered in the works, are valid for each layer.

It is seen from the material presented above that in the parametrization of a multilayer domain considered for each layer, and all the corresponding relations for a one-layered thin body under a new parametrization in [5–10], as well as for other parametrizations considered in [5–7, 11, 60], are hold under the condition that the root letters of quantities entering these relations must be equipped with the bottom index, which denote the number of the layer considered. In this connection, we do not consider the problems on the parametrization of a multilayer domain in detail. In what follows, if necessary, we write the necessary formulas from the corresponding relations of the works mentioned in this paragraph by the above method (equipping the root letters of quantities with the bottom index of the layer considered), and obtain some relations, which do not enter the above works.

1.4. Representation of the unit tensor of the second rank

It is easy to find this representation. Indeed, starting from the usual representation of this tensor [9, 10, 61, 62], by (28) and (29), we obtain the relation [1–5]

$$\underset{\sim}{\mathbf{E}} = \underset{\widetilde{\alpha}}{\mathbf{E}} = \underset{\alpha}{g}{}_{\tilde{p}}^{\breve{n}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\alpha}{\mathbf{r}}{}_{\breve{n}} = \underset{\widetilde{\beta}}{\mathbf{E}} = \underset{\beta}{g}{}_{\tilde{p}}^{\breve{n}}\,\underset{\beta}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}_{\breve{n}} = \underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}_{\breve{n}}, \quad \sim, \quad \smile\, \in \{-, \emptyset, +\}, \quad \forall \alpha, \beta, \tag{31}$$

which remains valid under index alternating. As is seen from (31), the quantities (26) and (27) introduced above represent the components of the unit tensor of the second rank (UTSR) for a multilayer thin domain of the three-dimensional Euclidean space. Now let us introduce the following definitions.

Definition 1.1 *The parametrization considered above, which is characterized by assigning the radius-vector of an arbitrary point of any layer α in the form* (1) *and by the fulfillment of relation* (3)*, is called the new parametrization of a multilayer thin domain.*

Definition 1.2 *The components* $\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}$, *for* $\tilde{p} \in \{\overset{-}{p},\, p,\, \overset{+}{p}\}$, $\breve{n} \in \{\overset{-}{n},\, n,\, \overset{+}{n}\}$, $\forall\, \alpha \neq \beta$, *and also the components* $\underset{\alpha}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}$, $\forall\, \alpha$, *for* $\sim\, \neq \smile$, *where* $\sim, \smile\, \in \{+, \emptyset, -\}$, *and the images obtained from them by index alternating are called the components of the unit second-rank tensor translation under the new parametrization of a multilayer thin domain.*

Definition 1.3 *The components* $\underset{\alpha\beta}{g}{}_{\tilde{p}\tilde{q}}^{\cdot\,\cdot}$, $\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\tilde{q}}$, $\underset{\alpha\beta}{g}{}^{\tilde{p}\tilde{q}}_{\cdot\,\cdot}$ *for* $\sim\, = -$ $(\sim\, = +)$, $\forall\, \alpha, \beta$, *and the components of the translation* $\underset{\alpha\beta}{g}{}_{\tilde{p}\breve{q}}^{\cdot\,\cdot}$, $\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{q}}$, *for* $\sim\, = +, \smile\, = -$ $(\sim\, = -, \smile\, = +)$, $\forall\, \alpha, \beta$, *are called the basic components of the second-rank unit tensor under the new parametrization of a multilayer thin domain if as as base surface, the inner (exterior) base surface of layers are taken.*

It is easy to find the expressions for $\underset{\alpha\beta}{g}{}_{pq}$ via basic translation components. Indeed, by (14), (26) and (29), we have

$$\underset{\alpha\beta}{g}{}_{pq} = \underset{\alpha}{g}{}_{p}^{\breve{m}} \underset{\beta}{g}{}_{q}^{\tilde{n}} \underset{\alpha\beta}{g}{}_{\breve{m}\tilde{n}} = (1 - x^3)^2 \underset{\alpha\beta}{g}{}_{\overset{-}{p}\overset{-}{q}} + x^3(1 - x^3)\big(\underset{\alpha\beta}{g}{}_{\overset{-}{p}\overset{+}{q}} + \underset{\alpha\beta}{g}{}_{\overset{+}{p}\overset{-}{q}} \big) + (x^3)^2 \underset{\alpha\beta}{g}{}_{\overset{+}{p}\overset{+}{q}}, \tag{32}$$

where $\sim, \smile\, \in \{-, +\}$, $\forall \alpha, \beta$. Whence, for $\alpha = \beta$, we obtain (15).

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