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Studying of the Proton Shell Evolution of Zr Isotopes within the Dispersive Optical Model

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Abstract—Proton single-particle spectra of ^{90,92,94,96,118,122}Zr isotopes are calculated using a mean field model with dispersive optical potential. The resulting single-particle energies ensure agreement between single-particle orbit occupation probabilities calculated using the formulas of the BCS theory, the available experimental data, and the atomic number Z. The difference between the calculated proton single-particle spectra and the magic-Z nucleus spectrum is described, along with effect the neutron structure of Zr isotopes has on the proton structure.

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INTRODUCTION

The evolution of the single-particle structure of nuclei when moving away from the β-stability valley is the subject of active studies in modern nuclear physics. Recent works show that the tensor component of nucleon–nucleon interaction, the dynamics of nucleus deformation, and the change in spin-orbital interaction play a great part in the evolution of nucleus structure upon a change in the number of nucleons over a wide range. Related studies have been performed using both microscopic and phenomenological approaches. The dispersive optical model (DOM) was first developed by Mahaux and Sartor [1]; it is a unified approach to detecting a complex semiphenomenological mean nuclear field at positive and negative energies. A technique has also developed for calculating shell potential by extrapolating some parameters established at positive energies to the region of negative values. Extrapolation is based on the use of dispersive equations that connect the real and imaginary parts of the mean field and effectively consider the correlations of a nucleon inside a nucleus. DOM was first used to describe the single-particle parameters of double-magic and magic spherical nuclei ^{40}Ca [1], ^{208}Pb [1], and ^{90}Zr [2–4]. The model was later extended to the region of unstable nuclei.

A technique for calculating the parameters of the dispersive optical potential (DOP) for stable and unstable spherical and nearby even–even nuclei was developed in [5, 6]. The technique is based on analyzing the experimental data on single-particle energies E_{nlj} and the population probabilities of single-particle orbits N_{nlj} for stable nuclei and then extrapolating the parameters to the region of unstable nuclei. During extrapolation, correspondence is achieved between number $Z(N)$ and the number of protons (neutrons) calculated using the Bardeen–Cooper–Schrieffer (BCS) theory for the population probabilities of single-particle orbits, while features of the Hartree–Fock component of DOP in the volume integral at $E = 0$ are taken into account. In this work, the technique suggested in [5, 6] is used to calculate the evolution of single-particle spectra of near spherical even–even Zr isotopes with $50 \leq N \leq 82$.

ANALYZING THE PROTON SINGLE-PARTICLE PARAMETERS OF STABLE ^{90,92,94,96}Zr ISOTOPES IN DOM

Even–even stable ^{90,92,94,96}Zr isotopes have relatively low values of their quadrupole deformation parameters ($\beta_2 \cong 0.1$), so a spherical DOM is used in calculations [2–4, 7–9]. The calculated E_{nlj}^{DOM} parameters were first compared with experimental values $E_{nlj}^{\exp}$ for ^{90}Zr in [2]. The experimental energy of the last mostly occupied proton state $2p_{1/2}$ was found to be $E_{-} = E_{2p_{1/2}}^{\exp} = -8.36$ MeV in [2], which virtually coincides with the energy of proton separation $S(N, Z)$ (taken with the opposite sign) from nuclei with numbers N and Z [10]. According to [2], the energy of the first primary free subshell $E_{+} = E_{1g_{9/2}}^{\exp} = -5.11$ or -5.16 MeV is close to the energy of proton separation $S(N, Z + 1)$ (taken with the opposite sign) from nuclei with the numbers N, $Z + 1$ [10]. A large particle-hole energy gap $\Delta_{1g_{9/2}-2p_{1/2}} = 3.2$ MeV is formed between these states, corresponding to the concept of the near magic properties of number $Z = 40$ for ^{90}Zr.

In view of this, proton energies E_{nlj}^{DOM}) were calculated in [2, 4] and good agreement with $E_{nlj}^{\exp}$ was

† Deceased.

achieved. However, the experimental energy values of the states $1g_{9/2}$ and especially $2p_{1/2}$ in the ^{90}Zr isotope, which are available in the literature, differ notably. Energies $E^{\exp}_{2p_{1/2}} = -7.03$ MeV and $E^{\exp}_{1g_{9/2}} = -5.72$ MeV found in [11] by analyzing data on one-nucleon proton transfer reactions correspond to the notably smaller gap $\Delta_{1g_{9/2}-2p_{1/2}} = 1.31$ MeV (see Table 1). In [7], the joint analysis of data on nucleon stripping and pickup reactions on a single nucleus [12] was used to measure experimental $E^{\exp}_{nlj}$ and $N^{\exp}_{nlj}$ values in stable Zr isotopes. These values for the ^{90}Zr nucleus are also given in Table 1. Compared to [11], the gap grew to 2.84(85) MeV, and $E_{2p_{1/2}} = -7.27(73)$ MeV was close to the data from [11]. In [7], the values of $E^{\exp}_{nlj}$ were compared to those of $E^{ДОМ}_{nlj}$ (see Table 1, row 7). Later on, $E^{\exp}_{nlj}$ and were corrected. As was noted in [8, 9], additional doubts arose when using the technique in [12] as a result of the incomplete experimental data on quantum parameters of the ^{89}Y levels at $E_x > 6.8$ MeV Another possible pattern of states (see Table 1, rows 9 and 10) was therefore given in [8, 9]. In row 12 of Table 1, the $E^{\rm DOM}_{nlj}$ values in [19] are compared to the $E^{\exp}_{nlj}$ values.

From the viewpoint of the additional criterion [5, 6] to correct the DOP parameters, the number of protons $N^{\exp}_{p}$ on proton subshells $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ is compared to the number of protons $N^{\rm BCS}_{p}$, in this work; the latter was found using the BCS equation

$$N^{\rm BCS}_{nlj} = \frac{1}{2}\left[1 - (E_{nlj} - E_{\rm F})\Big/\sqrt{(E_{nlj} - E_{\rm F})^2 + \Delta^2}\right], \quad (1)$$

where Δ is a gap parameter written as

$$\Delta = 0.25\left[2S(A,Z) - S(A+1, Z+1) - S(A-1, Z-1)\right] (2)$$

and equal to 1.122 MeV for ^{90}Zr when using the separation energy in [10]. Inserting energies $E^{\exp}_{nlj}$ [7] into Eq. (1) results in correspondence between the $N^{\rm BCS}_{p}\left(E^{\exp}_{nlj}\right)$ values (see Table 1, rows 5 and 6) and experimental data $E^{\exp}_{nlj}$. It was difficult to calculate $N^{\rm BCS}_{p}\left(E^{\exp}_{nlj}\right)$ with the $E^{\exp}_{nlj}$ data in [8, 9] due to their incompleteness.

To verify the proton DOP of an isotopic chain with number N varying over a wide range, it was suggested [5] that we control the value of the volume integral $J_{HF}(E = 0)$, since it was characterized by the property of approximate constancy (with a weak increase as the number N in the isotope grew). It turned out that this parameter corresponds to an overestimated value of the number $N^{\rm BCS}_{p}\left(E^{\rm DOM}_{nlj}\right)$ (see Table 1, row 11). The search for refined values of DOP parameters with allowance for the additional criteria in [5, 6] performed in this work allowed us to eliminate this inconsistency. The values of $E^{\rm DOM}_{nlj}$ and $N^{\rm BCS}_{nlj}\left(E^{\rm DOM}_{nlj}\right)$ according to data of this work are given in Table 1 (rows 12 and 13). According to Table 1 (row 12), the χ-squared test for the goodness of fit of $E^{\rm DOM}_{nlj}$ and $E^{\exp}_{nlj}$ [7] was around 1.5 times better than for $E^{\exp}_{nlj}$ [8, 9], and the corresponding number of protons was 12.0. The calculated gap $\Delta^{\rm DOM}_{1g_{9/2}-2p_{1/2}} = 2.91$ MeV also agreed with the data on $E^{\exp}_{nlj}$ [7].

Note that energy $E_{\rm F}$ is distributed such that energy difference $E_{1g_{9/2}} - E_{\rm F} = 1.90$ MeV for the calculated spectrum is considerably higher than difference $E_{\rm F} - E_{2p_{1/2}} = 1.0$ MeV. This is not typical of a classical magic number, so $Z = 40$ is not this number in ^{90}Zr. Nevertheless, this nucleus satisfies the magic criterion in other ways [9]. Note that the values of $N^{\rm BCS}_{nlj}(E^{\rm DOM}_{nlj})$ calculated in this work (Table 1, row 13) for states $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, and $1g_{9/2}$ are in good agreement with the values of $N^{\exp}_{nlj}$ according to [7]. We did not succeed in estimating the accuracy of $N^{\exp}_{nlj}$ for state $2p_{1/2}$ in [7]. Population probability $N^{\rm BCS}_{nlj}\left(E^{\rm DOM}_{nlj}\right)$ agrees with $N^{\exp}_{nlj}$ within an error of 20%. The DOP parameters calculated in this work for ^{90}Zr are given in Table 2 (row 2). All designations of the parameters correspond to [5, 6].

Table 3 presents values of $E^{\exp}_{nlj}$ and $N^{\exp}_{nlj}$ for ^{92}Zr taken from [11] (rows 2 and 3) and [8] (rows 4 and 5). Note the different values of gap $\Delta_{1g_{9/2}-2p_{1/2}}$ that correspond to the data in [8, 11]: $\Delta_{1g_{9/2}-2p_{1/2}} = 1.55$ MeV [11] and 2.68(161) MeV [8]. This difference is due to the greatly different values of energy $E^{\exp}_{1g_{9/2}}$, found in [11] and [8]. Note that no such great differences were observed for ^{94}Zr (Table 3, rows 8 and 10). In view of this, the available experimental data on ^{92}Zr are apparently in need of refinement. As with ^{90}Zr, the values of $E^{\rm DOM}_{nlj}$ calculated in [8, 9] were verified to ensure correspondence between proton numbers $N^{\rm BCS}_{p}\left(E^{\rm DOM}_{nlj}\right)$ on subshells $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$, and overestimates were found again. Recall that $\Delta_{1g_{9/2}-2p_{1/2}} =$ 1.55 MeV according to [11] and 2.68(161) MeV according to [8] for the ^{92}Zr isotope. Energies $E^{\rm DOM}_{nlj}$ and population probabilities $N^{\rm BCS}_{nlj}\left(E^{\rm DOM}_{nlj}\right)$, calculated in this work with gap parameter $\Delta = 1.265$ MeV are given in Table 3 (rows 6 and 7); the DOP parameters are given in Table 2 (row 3). Values $N^{\rm BCS}_{p}\left(E^{\rm DOM}_{nlj}\right) =$ 11.9 were obtained using these DOP parameters. Note that energies $E^{\rm DOM}_{nlj}$ for states $1f_{5/2,}$ $2p_{3/2}$, and $1g_{9/2}$

Table 1. Proton single-particle energies (in MeV) and population probabilities on valence subshells of ^{90}Zr

	$-E_{nlj}^{\exp}$	$N_{nlj}^{\exp}$	$-E_{nlj}^{\exp}$	$N_{nlj}^{\exp}$	$N_{nlj}^{\mathrm{BSC}}\left(E_{nlj}^{\exp}\right)$	$-E_{nlj}^{\mathrm{DOM}}$	$-E_{nlj}^{\exp}$	$N_{nlj}^{\exp}$	$-E_{nlj}^{\mathrm{DOM}}$	$N_{nlj}^{\mathrm{BSC}}\left(E_{nlj}^{\mathrm{DOM}}\right)$	$-E_{nlj}^{\mathrm{DOM}}$	$N_{nlj}^{\mathrm{BSC}}\left(E_{nlj}^{\mathrm{DOM}}\right)$
	[11]		[7]		n.c.	[7]	[8, 9]		[9]	n.c.	n.c.	
1	2	3	4	5	6	7	8	9	10	11	12	13
$1f_{7/2}$			15.56(156)	1.03	0.99	14.57			14.70	0.99	15.37	0.99
$1f_{5/2}$	9.87	0.96	10.43(104)	1.00	0.98	9.40	10.37(110)	1.00(2)	9.07	0.96	9.87	0.97
$2p_{3/2}$	9.27	0.90	10.11(105)	0.90(1)	0.97	9.35			9.32	0.97	9.27	0.96
$2p_{1/2}$	7.03	0.58	7.27(79)	0.68	0.71	7.88	6.97(70)	0.58(5)	7.34	0.86	7.76	0.83
$1g_{9/2}$	5.72	0.14	4.43(82)	0.08(1)	0.05	5.32	5.41(54)	0.06(5)	5.07	0.14	4.85	0.07
$2d_{5/2}$						0.30	−1.22(50)	0.03(2)			−0.5	
$\Delta_{1g_{9/2}-2p_{1/2}}$	1.31		2.84(85)			2.56	1.56(88)		2.27		2.91	
N_p		11.9		11.8(2)	11.7					12.8		12.0

Table 2. Proton DOP parameters for ^{90,92,94,96,118,122}Zr

DOP parameters	^{90}Zr	^{92}Zr	^{94}Zr	^{96}Zr	^{118}Zr	^{122}Zr
α_I, MeV fm^3	95.4	96.0	96.7	97.0	99.6	99.5
β_s, MeV	62.0	62.0	63.0	65.0	75.0	77.0
$-E_F$, MeV	6.75	7.72	8.56	9.48	18.40	19.60
$r_s = r_{\rm HF}$, fm	1.213	1.214	1.215	1.215	1.221	1.222
a_s, fm	0.664	0.664	0.664	0.664	0.660	0.660
r_d, fm	1.271	1.271	1.270	1.270	1.265	1.264
a_d, fm	0.566	0.567	0.568	0.569	0.580	0.582
r_{so}, fm	1.041	1.042	1.043	1.044	1.053	1.055
r_C, fm	1.240	1.239	1.238	1.238	1.232	1.231
V_{so}, MeV fm^3	5.5	5.75	6.0	5.75	5.0	5.0
$V_{HF}(E_{\rm F})$, MeV	58.85	59.3	59.6	60.2	65.1	65.9
$J_{HF}(E_{\rm F})$, MeV fm^3	491.7	495.8	498.7	502.9	544.2	551.0
$J_{HF}(0)$, MeV fm^3	466.4	467.0	466.8	467.7	477.9	480.6

$\beta_I = 12.5$ MeV, $a_{HF} = 0.615$ fm, $a_{so} = 0.59$ fm, $\gamma = 0.46$ for all isotopes.

agree with the experimental data within their errors. The differences are greatest for state $2p_{1/2}$.

The values of $E_{nlj}^{\rm exp}$ and $N_{nlj}^{\rm exp}$ for the ^{94}Zr isotope [8, 11] are given in Table 3 (rows 8, 9, and 10, 11, respectively). Population probabilities $N_{nlj}^{\rm BCS}\left(E_{nlj}^{\rm DOM}\right)$ calculated using data on $E_{nlj}^{\rm exp}$ from [8] and Eq. (1) with gap parameter $\Delta = 1.304$ MeV are given in Table 3 (row 12). The correspondence of $N_p^{\rm BCS}\left(E_{nlj}^{\rm exp}\right)$ to the number of protons on subshells $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ according to the data on $N_{nlj}^{\rm exp}$ indicates agreement in determining the values of $E_{nlj}^{\rm exp}$ and $N_{nlj}^{\rm exp}$ measured in [8]. The values of $E_{nlj}^{\rm DOM}$ calculated in this work (the DOP parameters in Table 2, row 4) and $N_{nlj}^{\rm BCS}\left(E_{nlj}^{\rm DOM}\right)$ are given in Table 3 (rows 13 and 14). The $E_{nlj}^{\rm DOM}$ values correspond fully to the $E_{nlj}^{\rm exp}$ values in [11, 8]. The calculated value of $N_p^{\rm BCS}\left(E_{nlj}^{\rm DOM}\right)$ also corresponds to the number of protons on the four valence subshells of ^{94}Zr found from the $N_{nlj}^{\rm exp}$ data. It should be emphasized that the good agreement between $E_{nlj}^{\rm DOM}$ and $N_{nlj}^{\rm BCS}\left(E_{nlj}^{\rm exp}\right)$ and the $E_{nlj}^{\rm exp}$ and $N_{nlj}^{\rm exp}$ data ([11] and [8]), respectively, confirms the assumption that we need to refine the $E_{nlj}^{\rm exp}$ and $N_{nlj}^{\rm exp}$ values for ^{92}Zr.

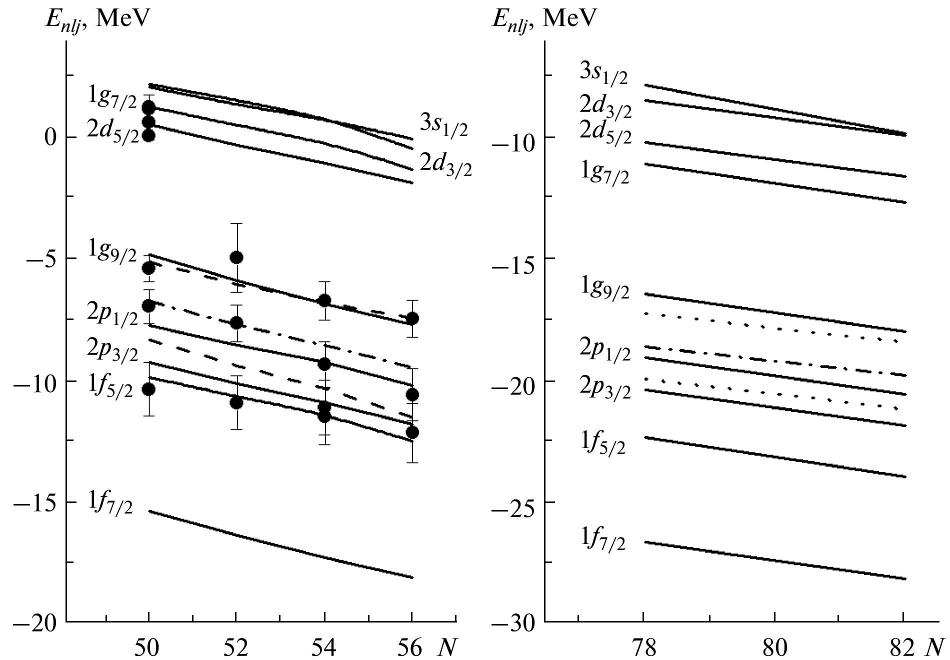
The data on $E_{nlj}^{\rm exp}$ and $N_{nlj}^{\rm exp}$ for the ^{96}Zr isotope were taken from [8] (Table 3, rows 15 and 16). The corresponding gap $\Delta_{1g_{9/2}-2p_{1/2}}^{\rm exp} = 3.11(130)$ MeV is larger than the one for ^{90,92,94}Zr, allowing us to assume [8, 9] that this proton gap in ^{96}Zr is affected by the possible closure of neutron subshell $2d_{5/2}$. The values of $E_{nlj}^{\rm DOM}$ calculated usingthe DOP parameters from Table 2 lead to gap $\Delta_{1g_{9/2}-2p_{1/2}}^{\rm DOM} = 2.52$ MeV, which is close to the calculated gaps in single-particle proton spectra of $^{90,\,92,\,94}$Zr, while population probabilities $N_{nlj}^{\rm BCS}\left(E_{nlj}^{\rm DOM}\right)$ calculated using gap parameter $\Delta = 1485$ MeV result in agreement with the number of protons on the four valence subshells in ^{96}Zr.

The proton single-particle spectra of ^{90,92,94,96}Zr isotopes near $E_{\rm F}$ are shown in the figure. For the sake of clarity, proton separation energies $S_p(A, Z)$ and $S_p(A+1, Z+1)$ (with the opposite sign) from [10] and the corresponding values of $E_{\rm F}^S$ are shown as well. Solid curves connect these values, and the values of $E_{nlj}^{\rm exp}$ from [7, 8] are plotted. The figure shows that the $E_{1g_{9/2}}^{\rm exp}$ and $E_{1g_{9/2}}^{\rm DOM}$ values are grouped near the $S_p(A+1, Z+1)$, while the $E_{2p_{1/2}}^{\rm exp}$ and $E_{2p_{1/2}}^{\rm DOM}$ values are approximately 0.9 MeV deeper than Fermi energy $E_{\rm F}$. The values $E_{1g_{9/2}}$ and $E_{2p_{3/2}}$ for all four Zr isotopes are not distributed symmetrically with respect to those of $E_{\rm F}$ (as is observed for classical magic numbers) but asymmetrically. We should note, however, that energy gaps $\Delta_{1g_{9/2}-2p_{1/2}}^{\rm DOM}$ for ^{90,96}Zr are slightly greater than for ^{92,94}Zr, distinguishing ^{90,96}Zr from the others. In addition [9], the values of energies 2_1^+ are also considerably higher for ^{90,96}Zr than for ^{92,94}Zr.

Table 3. Proton single-particle energies (in MeV) and population probabilities on valence subshells of $^{92,94,96,118,122}Zr$

	^{92}Zr						^{94}Zr						
	$-E^{exp}_{nlj}$	N^{exp}_{nlj}	$-E^{exp}_{nlj}$	N^{exp}_{nlj}	$-E^{DOM}_{nlj}$	$N^{BSC}_{nlj}(E^{DOM}_{nlj})$	$-E^{exp}_{nlj}$	N^{exp}_{nlj}	$-E^{exp}_{nlj}$	N^{exp}_{nlj}	$N^{BSC}_{nlj}(E^{exp}_{nlj})$	$-E^{DOM}_{nlj}$	$N^{BSC}_{nlj}(E^{DOM}_{nlj})$
	[11]		[8]		n.c.		[11]		[8]		n.c.	n.c.	
1	2	3	4	5	6	7	8	9	10	11	12	13	14
$1f_{7/2}$					16.36	0.99						17.27	0.99
$1f_{5/2}$	10.91	0.97	10.93(110)	1.00(2)	10.68	0.96	11.30	0.97	11.49(115)	1.00(2)	0.96	11.37	0.95
$2p_{3/2}$	9.94	0.86			10.15	0.94	11.07	0.85	11.11(112)	0.87(5)	0.94	10.91	0.94
$2p_{1/2}$	7.89	0.48	7.66(77)	0.49(3)	8.57	0.78	9.22	0.68	9.37(94)	0.75(5)	0.76	9.25	0.73
$1g_{9/2}$	6.44	0.18	4.98(142)	0.08(5)	5.92	0.09	7.17	0.14	6.74(80)	0.09(5)	0.09	6.86	0.10
$2d_{5/2}$					0.34	0.01						1.07	0.04
$\Delta_{1g_{9/2}-2p_{1/2}}$	1.55		2.68(161)		2.65		2.05		2.63(123)			2.39	
N_p		12.0				12.0		12.0		11.9	11.9		11.9

	^{96}Zr				^{118}Zr		^{122}Zr		
	$-E^{exp}_{nlj}$	N^{exp}_{nlj}	$-E^{DOM}_{nlj}$	$N^{BSC}_{nlj}(E^{DOM}_{nlj})$	$-E^{DOM}_{nlj}$	$N^{BSC}_{nlj}(E^{DOM}_{nlj})$	$-E^{DOM}_{nlj}$	$N^{BSC}_{nlj}(E^{DOM}_{nlj})$	$-E_{nlj}$
	[8]		n.c.		n.c.		n.c.		[15]
	15	16	17	18	19	20	21	22	23
$1f_{7/2}$			18.10	0.99	26.75	0.99	28.25	0.99	31.0
$1f_{5/2}$	12.17(122)	0.94(5)	12.52	0.95	22.44	0.98	24.03	0.98	25.5
$2p_{3/2}$			11.80	0.92	20.48	0.93	21.96	0.93	23.0
$2p_{1/2}$	10.59(106)	0.81(5)	10.22	0.72	19.16	0.75	20.65	0.76	22.0
$1g_{9/2}$	7.48(75)	0.00(0)	7.70	0.12	16.54	0.09	18.07	0.09	19.3
$2d_{5/2}$			1.91	0.01	10.29		11.71		11.5
$\Delta_{1g_{9/2}-2p_{1/2}}$	3.11(130)		2.52		2.62		2.58		2.7
N_p				12.0		12.0		12.0	



Proton single-particle energies of Zr isotopes. Dots show experimental data, solid line, calculations with DOP, dashed and dotted lines, proton separation energies $S(A, Z)$ and $S(A + 1, Z + 1)$ [10, 13], respectively, and dashed-dotted line, Fermi energies.

CALCULATING SINGLE-PARTICLE ENERGIES OF PROTON STATES FOR $^{118}_{40}Zr_{78}$ AND $^{122}_{40}Zr_{82}$ ISOTOPES

The BCS formula for calculating $N_{nlj}^{BCS}\left(E_{nlj}^{DOM}\right)$ values can be used for the Hartree–Fock component of the DOP of unstable nuclei, for which there are no data on single-particle energies. However, it is known that a rise in the number of neutrons in Zr isotopes sharply increases their nonsphericity. Deformation parameter $\beta_2 \sim 0.3$–0.4 for $^{100,102}Zr$, making it difficult to use the spherical DOM in calculating the parameters of these nuclei. According to the data in [13, 14], however, deformation parameters β_2 for, e.g., $^{118-124}Zr$ isotopes are comparable to deformation parameter β_2 for $^{90-96}Zr$ as N continues to grow. In view of this, calculations of E_{nlj}^{DOM} and N_{nlj}^{BCS} were performed only for $^{118,122}Zr$ using the spherical DOM in this work. The calculated DOP parameters are given in Table 2; the values of E_{nlj}^{DOM} and $N_{nlj}^{BCS}\left(E_{nlj}^{DOM}\right)$, in Table 3 (rows 19–22) for $^{118,122}Zr$. Gap parameters $\Delta = 1.282$ MeV and $\Delta = 1.25$ MeV, determined using the data in [13], were used in calculating $N_{nlj}^{BCS}\left(E_{nlj}^{DOM}\right)$ for ^{118}Zr and ^{122}Zr, respectively.

Energy E_{nlj} was calculated using the Hartree–Fock–Bogolyubov model with Gogny forces for the ^{122}Zr nucleus [15] with magic number of neutrons $N = 82$ [14]. Energies E_{nlj}^{DOM} agree with these results within limits of ≤10%. Correspondence is also observed in the values of energy gaps $\Delta_{1g_{9/2}-2d_{5/2}}^{DOM} \cong 6.4$ MeV. Differences between $E_{1p_{1/2}}^{DOM}$ and $E_{1g_{9/2}}^{DOM}$ for $^{118,122}Zr$ isotopes from the energies $-S(A,Z)$ and $-S(A+1, Z+1)$, respectively, show that $Z = 40$ is not a classic magic number for these isotopes either. The calculated energy of state $1f_{5/2}$ becomes stronger when the number of neutrons in Zr isotopes grows more rapidly than the energies of neighboring states. This results in the evolution of energy gaps between states $1f_{7/2}$–$1f_{5/2}$ (they are reduced) and $1f_{5/2}$–$2p_{3/2}$. In addition, the gap between states $2d_{5/2}$, $1g_{7/2}$ and $2d_{3/2}$, $3s_{1/2}$ grows.

CONCLUSIONS

The technique for calculating the DOP parameters of spherical and near spherical nuclei proposed in [5, 6] was used to study features of proton single-particle spectra of stable even–even isotopes $^{90,92,94,96}Zr$ and unstable isotopes $^{118,122}Zr$. This approach ensures agreement between the population probabilities of single-particle orbits near the Fermi energy $N_{nlj}^{BCS}\left(E_{nlj}^{DOM}\right)$ calculated using the BSC theory's formula with calculated energies E_{nlj}^{DOM} and experimental data N_{nlj}^{exp} found by jointly estimating the stripping and pickup reaction data for one nucleus. The calculated energies E_{nlj}^{DOM} and population probabilities $N_{nlj}^{BCS}(E_{nlj}^{DOM})$ correspond to incomplete population of subshell $2p_{1/2}$ in Zr isotopes. This shows that $Z = 40$ is not a classic (strong) magic number. Nevertheless, proton energy gap $\Delta^{\text{exp,DOM}}_{1g_{9/2}-2p_{1/2}}$ in stable Zr isotopes is quite high, especially in isotopes with the common magic number of neutrons $N = 50$ and new magic number $N = 56$. This result agrees with the increase in the sphericity observed in ^{90,96}Zr nuclei. The evolution of the calculated spectra of ^{118,122}Zr isotopes witnesses that the neutron structure affects that of protons.

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